Time and Repairing the Schwarzschild Solution

Introduction and relevance

In his theory of General Relativity, Einstein replaced the Lorentz transformation by the Minkowski formula. He also introduced the *variable* speed of light in gravitation which is in conflict with Noether's theorem on energy and momentum conservation.
Einstein's space-time must be placed within a wider *hypothetical* (Euclidean) Noether reference frame in either Cartesian or polar coordinates. Space-time curves within a Noether frame, light bends around mass but its speed is invariant, so all the constants of nature remain the same everywhere at all the time.

Einstein's variable speed of light in his General Relativity is untenable. By reinterpreting time as Schwarzschild did and using Shapiro's Solution, we can repair the Schwarzschild Solution.
This solves paradoxes, unites General with Special Relativity, excludes black hole singularities, describes cesium clocks accurately, and much more.

Minkowski, Shapiro, and the Schwarzschild radius

Let us first look at the formulas of Minkowski and Shapiro, and of the Schwarzschild radius:

$c^2.dt_0^2$	$= c^2.dt^2 - ds^2$	$[m_0^2]$	Minkowski formula	(1)
$c^2.dt_0^2$	$= (1 - R_S / r).c^2.dt^2 - dx^2 / (1 - R_S / r)$	$[m_0^2]$	Shapiro Solution	(2)
R_S	$= 2G.M / c^2$	[m]	Schwarzschild radius of the sun	(3)

In these formulas is "c" the speed of light, is "ds" the *space* distance ($ds^2 = dx^2 + dy^2 + dz^2$), is "$R_S$" the Schwarzschild radius, and is "G" the Newton constant. Applied to the sun: the Schwarzschild radius of the sun "R_S" is 2,953 meter, "M" is the mass of the sun, and "r" is the distance to center-of-mass of the sun. In the Shapiro experiment of (2) is "ds" equal to "dx", the reference frame is chosen such that the "y" and "z" remain zero.

Before we go further, we first need to understand time and time differences in the Shapiro and Schwarzschild Solution, as described by Karl Schwarzschild in 1916.

Reinterpreting Time in the Shapiro Solution

Time is measured on a (cesium) clock. Cesium clocks are influenced by speed and gravitation, as proven by the Hafele-Keating experiment. We know that "dt_0" is the *proper time* measured by the proper observer. But who measures "dt" in formula (2)? Karl Schwarzschild defined the time as the time measured from far, theoretically from infinity, which is independent of local gravitation.
We call this time difference as measured from far "dt_∞". Which means that the "dt" of formula (2) is better described as "dt_∞":

$c^2.dt_0^2$	$= (1 - R_S / r).c^2.dt_\infty^2 - dx^2 / (1 - R_S / r)$	$[m_0^2]$	Shapiro Solution with "dt_∞"	(4)

The Minkowski formula (1) is only valid in a Euclidean frame, so "dt" is the time difference within a (Euclidean) Noether frame. Within the Noether frame is ds = dx = c.dt, based on a constant speed of light within the Noether frame.

Based on formula (4) and the fact that a radar signal has a "dt_0" of zero, we get:

$dt_\infty = \mathrm{dt} / (1 - R_S / r)$ [s] Shapiro Solution (5)

The time difference as measured from infinity "dt_∞" is slightly longer than the synchronized time difference we use in the Noether frame "dt". So the Shapiro delay is caused by the denser space around the sun. Mass creates space, proven by Shapiro.
When the time difference between "dt_∞" and "dt" is integrated over the whole trajectory (over "x", while ds = dx), we obtain the "Shapiro delay".

Repairing Schwarzschild's Solution

We can now rewrite formula (4) by the substitution of (5) as:

$c^2.dt_0^2 = \{c^2.\mathrm{dt}^2 - \mathrm{ds}^2\} / (1 - R_S / r)$ $[m_0^2]$ re-interpreted Shapiro Solution (6)

Since ds = v.dt and $1 / \gamma^2 = 1 - v^2 / c^2$, this can be further reduced to:

dt_0^2	$= \mathrm{dt}^2 / \{(1 - R_S / r).\gamma^2\}$	$[s_0^2]$		
dt_0	$= \mathrm{dt} / \sigma.\gamma$	$[s_0]$	re-interpreted Shapiro Solution	(7)
σ^2	$= 1 - R_S / r$	[ ]	gravitation-factor squared	(8)

In these formulas is "σ" the gravitation-factor, the square root of the gravitational potential. According to Einstein's relativity principle, we can choose any other reference frame as long as the proper time difference "dt_0" remains the same. In other words, the solution is not just valid for the "x" direction, but we could also define the experiment in the "y" or "z" direction.

This means that we have found the *Schwarzschild Solution* based on a generic "ds"! This repaired solution does not just unite the Schwarzschild Solution with the Shapiro Solution and with Noether's theorem, but also with Einstein's theory of Special Relativity.

To see the difference with the current Schwarzschild Solution, let us put formula (6) in polar coordinates, $ds^2 = dr^2 + r^2.d\theta^2 + r^2.\sin^2.d\varphi^2$:

$c^2.dt_0^2 = \{c^2.\mathrm{dt}^2 - dr^2 - r^2.d\theta^2 - r^2.\sin^2.d\varphi^2\} / (1 - R_S / r)$

Let us also call $c^2.dt_0^2$ "ds^2" and replace "dt" by "dt_∞":

$ds^2 = (1 - R_S / r).c^2.dt_\infty^2 - dr^2 / (1 - R_S / r) - r^2.\{d\theta^2 + \sin^2.d\varphi^2\} / (1 - R_S / r)$ (9)

The red part is the difference with the current Schwarzschild Solution, which is used by many astronomers and lacks the red term. Look how much simpler formula (7) is compared to formula (9)!

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Repaired Schwarzschild's Solution

You may ask yourself, so what? Well, here it comes,

The Repaired Schwarzschild Solution:

1. is united with Special Relativity,
2. adheres to Noether's theorem,
3. excludes black hole singularities ($R > 1.5R_S$) based in the gravitation-factor inside a sphere,
4. describes the behavior of (cesium) clocks accurately,
5. allows you to compute the core temperature of the sun and the planets,
6. allows you to compute the escape speed from any location on earth (based on $E = \sigma.\gamma.m_0.c^2$),
7. simplifies the current solution, see formula (7), which is the short form of formula (9).

Summary

By interpreting time as Karl Schwarzschild did, and by defining a Noether frame in which energy and momentum is conserved, we get the Shapiro solution. It describes the difference between time differences as measured from far and the time differences you need to use within the Noether frame. The relativity principle then leads us to the repaired Schwarzschild Solution, which is much simpler and adheres to Noether's theorem.

Want to know more?

The book "Repairing Schwarzschild's Solution" describes it all in more detail.

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