Gravitational Potential "Φ_{in}" within a Sphere

Introduction and relevance

Einstein's theory of General Relativity (GR) is a complicated set of tensor operations aiming to make solutions independent of the coordinate system chosen. The authors want to take you back to the point where Einstein connected the tensor mathematics to real physics, which is described in the paragraph 21 of his entry "The Foundation of the General Theory of Relativity" in the "Annalen der Physik 49" of 1916. We will determine the gravitational potential within the sphere based on Einstein's Laplace operator, not based on Einstein's field equations as Karl Schwarzschild attempted and failed to do.

Surprising result: In real physics the radius of a sphere is always larger than its Schwarzschild radius and consequently*, a so-called "event horizon" does not exist*.

We will look at Einstein's gravitational potential as he described in paragraph 21, formulas (68) and (69). Then, based on the continuity of the gravitational potential and its first derivative, we will establish the gravitational potential *inside of* the sphere of incompressible liquid. We will check this outcome against the Laplace operator outcome as defined by Einstein. Finally, the authors will present you with the repaired Schwarzschild Solution based on Einstein's gravitational potential and its Laplace operator, a solution which also abides by Noether's laws of energy and momentum conservation.

Einstein's gravitational potential and Laplace operator

Einstein's gravitational potential outside of the sphere "Φ_{out}" (= g_{44} now called g_{00}) and its Laplace formula, see paragraph 21 formula (68) and (69), are given by:

$\Phi_{out} = 1 - 2G.M / c^2.r$	$r > R$	[ ]	grav. potential outside sphere (g_{00})	(1)
$\Delta\Phi = 8\pi.G.\rho / c^2 = 0$	$r > R$	[m^{-2}]	Laplace operator (Einstein)	(2)

In these formulas is "G" the Newton constant, "M" the mass of the sphere, "c" the invariant speed of light, "r" the distance to the center-of-mass, "R" the radius of the sphere, and "ρ" the density of the immediate surroundings ($\rho = 0$ outside of the sphere). We will use these formulas to determine the gravitational potential "Φ_{in}" within a sphere of incompressible liquid, without having to resort to coordinate transformations!

Einstein's gravitational potential within a sphere

We look for a formula within a sphere, of which the gravitational potential is continuous at the surface, is also continuous in its first derivative to "r" at the surface, and is at its minimum at r = 0. We then get (for the derivation see book II):

$\Phi_{in} = 1 - 3G.M / c^2.R + G.M.r^2 / c^2.R^3$	$r \leq R$	[ ]	gravitational potential *inside* (g_{00})	(3)

This formula (3) can be checked against Einstein's Laplace of (2):

$\Delta\Phi_{in} = 8\pi.G.\rho / c^2 = 6G.M / c^2.R^3$	$r \leq R$	[m^{-2}]	Laplace operator of "Φ_{in}"

In other words, formula (1) and (3) describe the gravitational potential for any “r” of a sphere of incompressible liquid, consistent with formula (2) and *obtained without any coordinate transformation*!

Consequences

This formula has a number of consequences that can be checked. Let us look at Einstein’s equations for a static situation (dx, dy, and dz are zero), also called a “time-like solution”:

$$ds^2 = c^2.dt_0^2 = g_{00}.c^2.dt_\infty^2 = \Phi.c^2.dt_\infty^2 \quad [m_0^2] \qquad \text{time-like solution} \qquad (4)$$

In this equation is “ds” the line-element, “dt_0” the proper time difference as measured by a proper observer, “g_{00}” an element of the covariant metric tensor (= gravitational potential Φ), and “dt_∞” the time difference as measured from far. When we call the square-root of “g_{00}” or “Φ” the gravitation-factor “σ”, we get:

$$dt_0 = \sigma.dt_\infty \quad [s_0] \qquad \text{behavior of clocks} \qquad (5)$$

The behavior of clocks on and above the earth is confirmed by many experiments (Hafele-Keating and others). Based on the Shapiro Solution (dt_∞ = dt / σ^2) and an invariant speed of light in all directions (Special Relativity, Minkowski formula, and the Michelson-Morley experiments), we get the repaired Schwarzschild Solution (see book II):

$$dt_0 = \sigma.dt_\infty = dt / \sigma.\gamma \quad [s_0] \qquad \text{repaired Schwarzschild Solution} \qquad (6)$$
$$E = m.c^2 = \sigma.\gamma.m_0.c^2 \quad [J] \qquad \text{repaired Schwarzschild Solution} \qquad (7)$$

The repaired Schwarzschild Solution is united with Special Relativity, with the Shapiro Solution and experiment, and with Noether’s conservation laws. This allows us to compute the core temperature of the sun and the planets, and the escape speed from any planet at any location. It also allows us to synchronize all cesium clocks on earth. And finally, looking at formula (3), we can state that the radius of any sphere is always larger than 1.5 times the Schwarzschild radius “R_S”, which equals 2G.M / c^2, based on a “Φ_{in}” larger than zero (clocks keep ticking at the center-of-mass where “r” is zero, see formula 5). *This means no “event horizon” in physics.*

More information?

Our three books (www.loop-doctor.nl) describe the repair of Einstein’s Relativity for Noether’s theorem in full detail. We hope you get as many “aha” experiences as we did,

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March 2018