# *Special Relativity based on Minkowski formula*

## Introduction and relevance

The theory of Special Relativity (SR) as published by Albert Einstein in 1905 is based on the Lorentz transformation. The accomplishments of SR are found in many areas: time dilation, energy in mass ($E = m.\mathrm{c}^2$), and the relativistic Doppler Effect.

However, Henrik Lorentz objected to the use of his transformation to two equal reference frames and Ehrenfest objected to the resulting deformation of a rigid cylinder (Ehrenfest paradox).

In 1908, Hermann Minkowski developed his own variation of SR based on a proper observer travelling within a dominant reference frame. This formula differs from the Lorentz transformation in two ways: 1) the proper observer has a very limited reference frame, which is subservient to the larger reference frame, and 2) the speed of the proper observer can vary, it is a differential formula.

Albert Einstein did not use the Lorentz transformation as the basis of his theory of General Relativity (GR), but chose to use Minkowski's differential formula. In other words, Einstein must have listened to both Lorentz and Ehrenfest. This article is about the changes in SR caused by the change of formula to Minkowski's formula. This change solves the twin, clock, and Ehrenfest paradox of SR.

## Differential Minkowski formula of SR

The formula is as follows:

$ds^2 = \mathrm{c}^2.dt_0^2 = \mathrm{c}^2.\mathrm{dt}^2 - \mathrm{dx}^2 - \mathrm{dy}^2 - \mathrm{dz}^2$ $[m_0^2]$ Minkowski space (1)

In this formula is "*ds*" the line element, "c" the speed of light, "$dt_0$" the time difference as measured by a travelling proper observer, "dt" the time difference on the synchronized clocks within the larger Euclidean reference frame, and are "dx", "dy", and "dz" the movements of the proper observer in time "dt". For example, a proper observer moving within the Noether frame (t,x,y,z) travels with a speed "v" within this Euclidean reference frame. The distance traveled "ds" (not italic) equals:

$\mathrm{ds}^2 = \mathrm{dx}^2 + \mathrm{dy}^2 + \mathrm{dz}^2$ $[\mathrm{m}^2]$ distance travelled (2)

The proper observer thus travels distance "ds" in time "dt" with speed "v" or ds = v.dt. This means for the Minkowski formula:

$\mathrm{c}^2.dt_0^2 = \mathrm{c}^2.\mathrm{dt}^2 - \mathrm{ds}^2 = (\mathrm{c}^2 - \mathrm{v}^2).\mathrm{dt}^2$ $[m_0^2]$ Minkowski space

With the help of the boost-factor "γ", which equals:

$\gamma = (1 - \mathrm{v}^2 / \mathrm{c}^2)^{-½}$ [ ] boost-factor (3)

we get:

$dt_0 = \mathrm{dt} / \gamma$ $[s_0]$ time dilation (4)

The resulting time dilation is the same as the outcome of the Lorentz transformation, *if* the proper observer is located in the origin of S' and S is a Euclidean reference frame.

## Difference between Lorentz transformation and Minkowski formula

The Minkowski formula differs in two principle ways from the Lorentz transformation:

1. Minkowski formula is a relation between a single proper observer and observers of a Euclidean reference frame, while the Lorentz transformation as was used by Einstein in 1905 is based on two equal reference frames,
2. The clock and time of the proper observer are influenced by its speed within the reference frame in the Minkowski formula, while the synchronized clocks are not influenced by the clock of the proper observer. The Lorentz transformation as used by Einstein results in a slower clock and time of the *other* reference frame creating the twin and clock paradox.

The first problem that Einstein encountered in developing his theory of General Relativity is that reference frames cannot be equal. According to his own theory as published in 1916, he states (translated) following formula (72):

"Thus the clock goes more slowly if set up in the neighborhood of ponderable masses".

In other words, Einstein recognizes that proper clocks in gravitation tick slower than clocks far away from the massive objects. The progress of physics within a small reference frame around a proper clock (the proper frame) proceeds slower than within a Euclidean frame far away from that same mass. This demonstrates that reference frames cannot be equal; the more mass, the more influence. To physicists this is not an issue, cesium clocks indicate time and two clocks cannot both go faster and slower at the same time. The slower clocks in stronger gravitation are confirmed by many experiments (Hafele-Keating, Pound-Rebka, and GPS satellites).

The second problem he encountered was the Lorentz transformation. He needed his theory of Special Relativity to be valid at an infinitely small reference frame in the neighborhood of a ponderable mass. The Lorentz transformation is valid at a known starting point and a constant speed. In contrast, the Minkowski formula is valid at any point in time and at any speed, even at a variable speed. So, as basis for his theory of General Relativity, he used the Minkowski formula, not the Lorentz transformation for that infinitely small reference frame.

## Solving the clock and twin paradox

The clock and twin paradoxes are solved by the acceptance that the proper clock is influenced by the reference frame of a ponderable mass, like the earth and not vice versa. In other words, the muons created in our atmosphere by the solar wind have a clock and time that ticks much slower than the clocks on earth. The proper observers of these muons also recognize the faster clocks and time of the observers standing still on earth.

The dominance of a reference frame depends on its contained mass and is measured on the clock; the faster the clock the more dominant the reference frame is. The reference frame around a muon is subservient to the reference frame of the earth. The universal frame is the ultimately dominant reference frame. This is not a return to the ether, it merely states that universal clocks tick fastest.

The twin that travels through the universal frame at high speed gets time dilated “$dt_0$” according to formula (4). The twin that stays behind with time difference “dt” within the dominant universal frame gets old much quicker. So yes, upon return, the travelling twin is much younger than its twin on earth.

## Solving the Ehrenfest paradox

The Ehrenfest paradox describes the deformation of a rigid cylinder. A fast rotating and rigid cylinder is deformed according to the Lorentz transformation since the high-speed outside of the cylinder gets length contracted, while the radius of the sphere remains the same. It would mean that the circumference of a circle is no longer 2π times its radius!

SR according to the differential Minkowski formula (1) does not have that problem, equal length, width, and height contraction. In other words, the high-speed cylinder does not deform and its circumference remains exactly 2π times its radius. In other words, the Minkowski formula solves the Ehrenfest paradox of SR.

## Conclusion

Einstein invalidated the equality of reference frame in his theory of General Relativity, contradicting Special Relativity. He also used the Minkowski formula as a basis of General Relativity, indirectly admitting its superiority over the Lorentz transformation. In a theory of Special Relativity based on the Minkowski formula, its paradoxes disappear. For more information, see www.loop-doctor.nl.

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Schiedam, March 2018