Einstein's change of mind on Special Relativity

Introduction and relevance

Einstein published his first version of the theory of Special Relativity in 1905 (SR1905). Without saying as much, he altered his own SR1905 into a version suitable for his theory of General Relativity (GR), which he published in 1916, which we will call the theory of Special Relativity of 1916, in short: SR1916.

This article is about the who, why, and how of Einstein's change of mind. At first we will look at the problems with SR1905. Then we will look at the alternative offered to him by his former teacher Minkowski. Finally, we will look at SR1916, which has no paradoxes and serves as a good foundation of his GR.

Problems with SR1905

The SR1905[1] is based on the Lorentz transformation of two reference frames K and K' that speed away from each other at a constant speed "v". The *assumption* is that physics can be applied in both reference frames *simultaneously*. The Lorentz transformation from (t,x,y,z) to (t',x',y',z') is:

t'	$= \gamma.(t - v.x / c^2)$	coordinate t' in K' is function of coordinates t and x in K	(1)
x'	$= \gamma.(x - v.t)$	coordinate x' in K' is function of coordinates t and x in K	(2)
y'	$= y$	coordinate y' in K' equals coordinate y in K	(3)
z'	$= z$	coordinate z' in K' equals coordinate z in K	(4)
γ	$= (1 - v^2 / c^2)^{-½}$	boost-factor	(5)

The origins of both reference frames at time zero are at the same space-time location (0,0,0,0) and the reference frames overlap each other. For example, if v = 0.6c, in which "c" is the speed of light, then the boost-factor "γ" is equal to 1.25. When x = v.t, then is:

$t' = t / \gamma = t / 1.25$ time dilation of t' in K' relative to t in K

In other words, the time t' indicates only 4 seconds while time t indicates 5 seconds. However, based on historically, mistakenly assumed equality of reference frames, K' could claim to stand still while K is travelling, in which case we see time dilation of t in K. Which clock ticks slowest? This is the essence of the clock paradox and twin paradox.

There is another problem, Ehrenfest pointed out that a rigid cylinder which rotates at a very high speed, would result, based on the Lorentz transformation, in a lower circumference ($x' = \gamma.(x - v.t)$, length contraction), while the radius remains exactly the same ($y' = y$, no width contraction). How can a rigid cylinder have a lower circumference and the same radius? This is the Ehrenfest paradox.

Furthermore, Mach objected to the equality of reference frames. Mach argued that the inertia of mass is relative to all other masses in the universe. This means that the reference frame of the universe is a dominant reference frame and that the reference frame of the Earth is dominant over small particles.

In summary, SR1905 presents us with unsolvable paradoxes.

[1] Einstein A, "On the electrodynamics of moving bodies" 1905

Remarkably, SR1905 is still taught to students in spite of Einstein's changes of mind in 1916, even though the alternative is well described!

Minkowski offers an alternative

The Minkowski formula is a very different way of looking at SR. Minkowski was Einstein's professor and his formula does not relate two reference frames to each other, but relates a single particle (mass-point or photon) to a Euclidean (flat or straight) reference frame. The Minkowski formula is differential and goes:

$$ds^2 = c^2.dt_0^2 = c^2.dt^2 - dx^2 - dy^2 - dz^2 \quad [m_0^2] \quad \text{Minkowski formula} \qquad (6)$$

In this formula is "*ds*" the line-element, "dt_0" the time difference as measured by a proper observer (moving along) the particle, and are "dx", "dy", and "dz", the change in coordinates over time "dt". The Minkowski formula also offers time dilation, given that $ds^2 = dx^2 + dy^2 + dz^2$:

$$c^2.dt_0^2 = c^2.dt^2 - ds^2 \quad [m_0^2] \quad \text{Minkowski formula} \qquad (7)$$

Note that "ds" is a *space* distance and that "*ds*" is the line-element, a measure of the proper time difference "dt_0" ($ds = c.\, dt_0$). Given that ds = v.dt in formula (7), we get:

$$c^2.dt_0^2 = c^2.dt^2 - v^2.dt^2 \quad [m_0^2] \quad \text{Minkowski formula}$$

Then based on formula (5) of the boost-factor, we get:

$$dt_0 = dt / \gamma \quad [s_0] \quad \text{time dilation of the proper observer}$$

In other words, the time "dt_0" indicates 4 seconds while time "dt" indicates 5 seconds, time dilation of the proper observer, just like the Lorentz transformation. The Minkowski formula is quite different from the Lorentz transformation: 1) the speed "v" may vary, 2) there is a dominant reference frame, and 3) there is no difference between contraction in the x and y/z directions.

Einstein's change of mind in 1916

The criticism of Mach and Ehrenfest and the flexibility of the Minkowski formula have probably led to Einstein's changed of mind. He writes in 1916 in paragraph 3 about the Lorentz transformation[2]:

"….Hence Euclidean geometry does not apply to K'…."

Not being Euclidean would mean that the laws of physics including geometry would no longer be applicable! Einstein acknowledges the criticism of Ehrenfest about his SR1905.The very first formula in this 1916 publication[3] is about SR and Minkowski's improvement:

"By the Special theory of Relativity the expression

$$ds^2 = -dX_1^2 - dX_2^2 - dX_3^2 + dX_4^2 \qquad (1)$$

[2] Einstein A, "The Foundation of the General Theory of Relativity", Annalen der Physik" 1916 paragraph 3
[3] Einstein A, "The Foundation of the General Theory of Relativity", Annalen der Physik" 1916 paragraph 4

then has a value which is independent of the orientation of the local system of co-ordinates and is ascertainable by measurements of space and time."

In fact, he abandons the limited Lorentz transformation. So his very first formula of GR describes the renewed SR1916 by the Minkowski formula, in which $X_1 = x$, $X_2 = y$, $X_3 = z$, and $X_4 = t$, a formula between a proper observer and a Euclidean reference frame, no longer using the Lorentz transformation. Finally, to obliterate his own SR1905, he states in paragraph 22 of the same document[4]:

"Thus the clock goes more slowly if set up in the neighborhood of ponderable masses"

This takes away the last straw of his original SR, the equality of reference frames, the reference frame with the most mass determines the clock speed of the (proper) reference frame with less mass, answering to the criticism of Mach. As a result, all paradoxes have disappeared.

The myths about SR keep lingering on for those that have never read his GR. For those that really have read GR, the twin, clock, ladder, and Ehrenfest paradoxes disappear like snow under the sun. For those that have studied Relativity, the Hafele-Keating experiment, and Muons (created by the solar wind) reaching the surface, prove that reference frames are not equal. The clocks in high-speed planes tick slower than the clocks standing still on earth. The clock and time of the muon ticks much slower than the clocks on earth. Goodbye twin and clock paradox.

Special Relativity according to Einstein in 1916

In Einstein's theory of Special Relativity of 1916 as described in his final document on GR, there is no twin, clock, ladder, or Ehrenfest paradox. The speed of light is the invariant "c" to all observers. The correct formula is the Minkowski formula, not the Lorentz transformation. Time dilation is valid for a mass-point with a proper observer moving within a Euclidean reference frame. There is simultaneous length-, width-, and height-contraction.

Final remarks: the Minkowski formula adheres to Noether's theorem of energy and momentum conservation, while the Lorentz transformation does not, see our book "Repairing Special Relativity". Einstein's change of mind is what a real scientist does: change your theory when overwhelming evidence of errors in your theory is presented, while an alternative is available.

More information?

Our three books (www.loop-doctor.nl) describe the repair of Einstein's Relativity for Noether's theorem[5] in full detail. We hope you get as many "aha" experiences as we did,

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[4] Einstein A, "The Foundation of the General Theory of Relativity", Annalen der Physik" 1916 paragraph 22

[5] Noether E. "Invariant variation problems" translated by Tavel M. TTSP 1971 p. 186-207