Covariant metric tensor of Schwarzschild's Solution

Introduction and relevance

Contrary to popular believe, there are many Schwarzschild Solutions. Karl Schwarzschild found two solutions: to a mass-point in January 1916[1] and to a sphere of incompressible liquid in February 1916[2]. Einstein came out with his final publication of General Relativity[3] after Karl's death in 1916. He made an important change to his earlier documents in the "Annalen der Physik" of 1915, on which Karl had based his solutions. The Dutchman J. Droste was the first to find a solution to a mass-point[4] based on Einstein's official document. This solution *varied slightly* from Karl's mass-point solution. Arthur Eddington *copied* the solution of J. Droste of a mass-point[5].

Misner, Thorne, and Wheeler (MTW) *copied* Eddington's solution to a mass-point and *expanded* the solution to a sphere of incompressible liquid[6], which differs substantially from Karl's original. Many authors of Relativity this century copy the MTW solution, which is described in this document as "the" Schwarzschild Solution. Kip Thorne of MTW went on with "Eddington-Finkelstein" coordinates to "fall through the event horizon" of a collapsing black hole[7].

The authors will show that *both* "the" Schwarzschild Solution of MTW/Eddington/Droste *and* Thorne's solution of 1994 are contrary to Einstein's theory of General Relativity (GR) of 1916. The authors eliminate the historic errors and unite this *repaired* Schwarzschild Solution with Noether's theorem of energy and momentum conservation. The *repaired* Schwarzschild Solution explains the Shapiro delay and solution, uncovers the "event horizon" as science fiction, and provides you with a minimal radius of a sphere of incompressible liquid. In other words, the Schwarzschild radius, the so-called "event horizon" is always smaller than the actual radius.

The repaired Schwarzschild Solution is not just uniting Noether's theorem with Einstein's Relativity, but it also unites Einstein's GR with his own theory of Special Relativity and with Minkowski's formula. Practical results include the computation of the core temperature of the earth and the sun, the escape speed from earth at different locations, and the science fiction of a black hole as singularity.

We are going to look at the conditions to coordinate system transformations as defined by Einstein in 1916, the historic errors which keep on being copied over and over again, the strange semantics used by authors to get to the *presumed* "correct" outcome. We are also going to look at Noether's theorem to understand energy and momentum conservation, look at the different time differences as measured by different observers, and look at Shapiro's delay and solution to find a relation between the different time differences.

We will end with a definition of the covariant metric tensor, which is consistent with Einstein's demands and abides by Noether's theorem as well. It is the ultimate merger of the *physics of Noether's theorem* and the *mathematics of Einstein's Relativity*!

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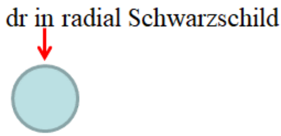
[1] Schwarzschild, K (1916), "On the Gravitational Field of a Mass-point"
[2] Schwarzschild, K (1916), "On the Gravitational field of a Sphere of incompressible Liquid"
[3] Einstein A. (1916) "The General Theory", Annalen der Physik, 49
[4] Droste J. KNAW proceedings 19 I (1917) page 199.
[5] Eddington, A (1922), "The Mathematical Theory of Relativity"
[6] Misner, Thorne, and Wheeler (1970), "Gravitation"
[7] Thorne, K (1994), "Black Holes and Time Warps, Einstein's Outrageous Legacy"

Covariant metric tensor of Schwarzschild’s Solution

Conditions to coordinate system transformations

The covariant metric tensor “g_{ii}” of Einstein’s theory of General Relativity (GR) determines the geometry of space-time at a location. The space-time location is independent of the coordinate system or units chosen, the essence of the relativity principle. In figure 1 the covariant metric tensor is given in red for two proven solutions (radial Schwarzschild and Shapiro), for light in vacuum:

$$c^2.dt_0^2 = 0 = \begin{bmatrix} c.dt_\infty & dr & 0 & 0 \end{bmatrix} \begin{bmatrix} \sigma^2 & 0 & 0 & 0 \\ 0 & -\sigma^{-2} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} c.dt_\infty \\ dr \\ 0 \\ 0 \end{bmatrix}$$



$$c^2.dt_0^2 = 0 = \begin{bmatrix} c.dt_\infty & dx & 0 & 0 \end{bmatrix} \begin{bmatrix} \sigma^2 & 0 & 0 & 0 \\ 0 & -\sigma^{-2} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} c.dt_\infty \\ dx \\ 0 \\ 0 \end{bmatrix}$$

$\sigma^2 = 1 - R_S / r$

$g = -1$

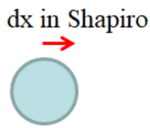


Figure 1: covariant metric tensor g_{ii} for light in vacuum (in red)

The proper time difference “dt_0” for light is zero. The often quoted line element or space-time distance “ds” equals $c.dt_0$. We will use the symbol “ds” for space distance only, as Riemann did. The figure shows the use of polar coordinates (t,r,θ,φ) in the Schwarzschild solution in radial direction, in which “dθ” and “dφ” are zero. The figure also shows the use of Cartesian coordinates (t,x,y,z) by Shapiro in a single direction “dx”. The covariant metric tensor is independent of the coordinate system chosen these are the same “g_{ii}” (in red). Note that both in the radial Schwarzschild Solution and in the Shapiro Solution the time is measured from far (infinity) and symbolized by “dt_∞”.

In figure 1 is “σ” the gravitation-factor, the square root of Einstein’s gravitational potential “Φ”, in which $\Phi = \sigma^2 = 1 - R_S / r$. The Schwarzschild radius “R_S” equals $2G.M / c^2$, in which “G” is the Newton constant, “M” the mass of a sphere, and “c” the speed of light. The Schwarzschild radius of the earth equals about 8.88 [mm]. The determinant of the covariant metric tensor “g” equals minus one, $g = g_{00}.g_{11}.g_{22}.g_{33} = -1$,which Einstein symbolized by $g = -1$.

However, not all coordinate system transformations are allowed to get meaningful results. Einstein used Cartesian coordinates, as did Shapiro, while Schwarzschild used polar coordinates, and Thorne used tortoise (Eddington-Finkelstein) coordinates. To transform coordinates in GR, we need to make sure that the result is the same measured proper time “dt_0” in the same proper units second [s_0], or the same distance “ds” ($ds = c.dt_0$) in proper meters [m_0].

That’s why Einstein attached conditions to coordinate system transformations used with the covariant metric tensor. Two conditions were defined in Einstein’s GR of 1915: $g = -1$, and the volume element must be equal within both coordinate systems. Karl Schwarzschild made use of $g = -1$ and the volume element in order to arrive at his two solutions (to a mass-point and to a sphere of incompressible liquid) in resp. January and February 1916.

Covariant metric tensor of Schwarzschild's Solution

Historic errors based on g = –1

The authors will show that Einstein's demand of g = –1 as originally described in "die Annalen der Physik" of 1915 is contrary to Noether's theorem. This demand leads to a failure of the speed of light in the transversal directions r.dθ and r.sinθ.dφ of the Schwarzschild Solution, as was proven by Shapiro's solution and experiment. More about that later. One year later, at the official publication of "The General Theory of Relativity", Einstein stated:

"On the abandonment of the choice of coordinates with g = –1, there remain *four* functions of space with liberty of choice, corresponding to the four arbitrary functions at our disposal in the choice of coordinates". It was in a *single footnote* in paragraph 19. In other words, he *abandoned* g = –1 as a *requirement* a year after Karl Schwarzschild found two exact solutions based on g = –1. Karl Schwarzschild died before the official publication, he could not correct his two solutions anymore! In fact, nobody corrected the Schwarzschild Solution after Karl's death. Amazingly, authors did the strangest things to make their Schwarzschild Solution *look like* the originals! Here are four examples:

Droste J. in 1917: "..,we are at liberty to choose instead of r a new variable which will be such a variable of r, that in ds^2 the coefficient of the square of its differential becomes unity"[8].
Eddington A. in 1922: "There is no reason to regard *r* in (38.12) as more immediate the counterpart of r in (38.11) than *r1* is… We shall here choose and accordingly drop the suffix,…"[9].
Misner, Thorne, en Wheeler in 1970: "…With this choice of the radial coordinate and with the primes dropped, equation (23.3) reduces to…."[10].
OAS G. in 2005: "In fact, we can limit this to W = X = 1 without loss of generality"[11].

These authors have used semantics to unify two variables that were based on g = –1, even though Einstein had abandoned this requirement! Did they not read Einstein's footnote in paragraph 19? Did they work towards the *presumed* correct outcome? Did they just copy an earlier version? In fact, *none of these authors have actually worked out the exact solution to either a mass-point or to a sphere of incompressible liquid as Karl Schwarzschild did.*

To make it worse, the relativity principle talks about coordinate *system* transformations, not about coordinate transformations; how do you mean "we are at liberty to choose" (Droste J.)? In mathematics, any coordinate can be transformed in many ways, but in physics the result has to be put back into a meaningful coordinate system! A failure to do so, as the examples above, results in a difference in the local (proper) time difference "dt_0"! That's why Einstein stated in paragraph 4:

"These can no longer be dependent on the orientation and the state of motion of the 'local' system of co-ordinates, for ds^2 is a quantity ascertainable by rod-clock measurements of point events infinitely proximate in space-time, and defined independently of any particular choice of co-ordinates".[12]

In other words, any coordinate *system* may be used as long as the proper (local) time difference "dt_0" on a (cesium) clock remains the same. When you do any coordinate transformation, these must

[8] Droste J. KNAW proceedings 19 1,1917 page 199.
[9] Eddington A. "The Mathematical Theory of Relativity", 1922 after formula (38.13).
[10] Misner, Thorne, and Wheeler, "Gravitation", 1970 between formula (23.6) and formula (23.7).
[11] Oas G. "Full derivation of the Schwarzschild Solution" Harvard Summer School (Pdf), 2005.
[12] Einstein A. "The General Theory", Annalen der Physik, 49, 1916 paragraph 4 formula (3) and following.

be reversible, such that these can be reversed to get the final outcome in the chosen coordinate system!

Interpreting Einstein's volume element (scalar) demand

In his final publication, Einstein explains the volume element in paragraph 8. In this paragraph, he talks about the "volume scalar" instead of the earlier "volume element" that Karl Schwarzschild used.
It is in this paragraph that Einstein explains why the determinant condition g = –1 was important. This volume scalar "dτ" is a multiplication of c.dt, dx, dy, and dz in [m^4] at an infinitely small region. When g = –1, then, according to formula (18a), the volume scalar remains the same after a coordinate system transformation.

Karl Schwarzschild used the earlier volume element as the multiplication of dx, dy, and dz in [m^3]. He argued that if you change from Cartesian to polar coordinates, the volume element in [m^3] remains the same:

$dx.dy.dz = r^2.dr.d\theta.\sin\theta.d\varphi$ [m^3] Schwarzschild's volume element

Why did Einstein change the volume element into a volume scalar in his final publication? And why did he abandon g = –1? Let us get back to see what Einstein's original intent was on which he based his g = –1 demand. The original intend is found in the relativity principle in paragraph 4, "ascertainable by rod-clock measurements" (see previous paragraph). He mistakenly translates this intend into g = –1 and corrects this mistake by abandoning this demand in paragraph 19.

However, that does not mean that every coordinate (system) transformation is allowed. The proof of the pudding is simple, the time difference on a local (proper) clock "dt_0" must remain the same after transformation! Good examples are: putting the origin of a Cartesian coordinate system at another physical location, like Shapiro did. Another good example is changing Cartesian coordinates into polar coordinates, like Karl Schwarzschild did.

Bad examples leading to a different "dt_0" are the coordinate transformations as described in the previous paragraph. Another bad example is the "tortoise coordinate system" of Thorne, which in no way can guarantee the same proper time difference "dt_0" at the (mistaken) "event horizon". The "event horizon" is in itself questionable, more about that later. Let us first repair the Schwarzschild Solution for Einstein's change of mind and for Noether's theorem.

To do that, we need to understand the difference between reference frames in physics and *measured* curved space-time first. Then we will repair the g = –1 condition for Noether's theorem. Finally, we will repair the Schwarzschild Solution for Noether's theorem. This repaired Schwarzschild Solution is both elegant in its outcome ($E = m.c^2$) as practical in its applications (core temperature of the earth and the sun based on energy conservation, for one).

Covariant metric tensor of Schwarzschild's Solution

Euclidean reference frames and curved space-time

Both Cartesian (x,y,z) and polar (r,θ,φ) coordinate systems are only useful in Euclidean space. Euclidean space is space in which Pythagoras is applicable. In curved space, Pythagoras is no longer applicable and the sum of three angles of a triangle is not 180 degrees. In order to identify a location, one needs Euclidean space and Pythagoras. How else could you identify a location if the space is curved? This is where confusion sets in with curved space. Einstein has demonstrated that space curves. How do we cope with coordinate systems in curved space?

This is where Emmy Noether steps in. We must distinguish between *Euclidean reference frame with coordinate systems* and *real (curved) space*. Emmy Noether defined the conditions to a reference frame under which energy and momentum conservation can be proven, we will call such a reference frame a "*Noether frame*". A Noether frame is Euclidean in space, with synchronized clocks, in which the laws of nature and its constants (like "c") are independent of location and time. The covariant metric tensor of Einstein describes the curvature of space-time within a Noether frame. In other words, *real space-time curves within a hypothetical Noether frame*.

General Relativity and the Schwarzschild Solution

Einstein modified Minkowski space-time to define curved space-time. The covariant metric tensor is a 4 x 4 matrix of 16 elements, which is symmetrical ($g_{ij} = g_{ji}$), leaving 10 elements to be unique. In static and symmetrical spheres of incompressible liquid (second Schwarzschild Solution), only four elements remain unequal to zero, namely g_{ii}, the other 12 elements are zero (these elements "vanish"). We can thus describe the Schwarzschild Solutions in terms of the curvature of space-time at a location within the (Noether) reference frame (t,r,θ,φ) as:

$$c^2.dt_0^2 = g_{00}.c^2.dt_\infty^2 + g_{11}.dr^2 + g_{22}.r^2.d\theta^2 + g_{33}.r^2.\sin^2\theta.d\varphi^2 \quad [m_0^2] \quad \text{Schwarzschild Solution (polar)}$$

Note that time "dt_∞" is measured from infinity, where the clock ticks at the fastest rate. Also note that in the Minkowski formula: $g_{00} = 1$, $g_{11} = -1$, $g_{22} = -1$, $g_{33} = -1$, which is the solution in the Newtonian limit. In many publications, the time difference measurement "dt_∞" is printed as "dt", but we need "dt" as the Noether time difference, which we will see later. Note that the Schwarzschild Solution, as a result of the "relativity principle", can also be written in Cartesian coordinates as (the coordinate transformation is found at the end of this article):

$$c^2.dt_0^2 = g_{00}.c^2.dt_\infty^2 + g_{11}.dz^2 + g_{22}.dy^2 + g_{33}.dx^2 \quad [m_0^2] \quad \text{Schwarzschild Solution (Cartesian)}$$

The key to find the four values of "g_{ii}" is the essence of General Relativity, the four "field equations" of the Schwarzschild Solutions. Karl Schwarzschild found both solutions based on Einstein's condition:

$$g = g_{00}.g_{11}.g_{22}.g_{33} = -1 \quad \text{Einstein's 1915 determinant condition}$$

He worked out "g_{00}" as $(1 - R_S / R)$, in which "R_S" is the Schwarzschild radius and "R" is an auxiliary variable ($R^3 = r^3 + R_S^3$). He found "g_{11}" as $1 / g_{00}$. He argued that $g_{22} = g_{33}$ based on the symmetry of the sphere. This leaves only one choice for g_{22} and g_{33}, namely to be equal to minus one. We now have the original Schwarzschild Solution for a *mass-point*; the outside of a sphere of incompressible liquid is the same, but with one difference: $R^3 = r^3 + \rho$. To get further, we need to look at time and how time is measured.

Covariant metric tensor of Schwarzschild's Solution

Time in Noether frames and in curved space-time

Time "t" in a Noether frame is as hypothetical as space. This time cannot be found on any clock, unless the real space is Euclidean. Any event within a Noether frame is thus identified as (t,x,y,z) in Cartesian coordinates or (t,r,θ,φ) in polar coordinates. However, there is no clock that indicates time "t" within curved space-time within a Noether frame. Time "t" is just as elusive as (x,y,z) in curved space-time. What we can do is measure time from far, as Schwarzschild did. However, we must recognize that a time difference as measured from far "dt_∞" differs from the Noether time difference "dt", as was proven in the Shapiro experiment, more about that later.

The locally measured time difference is called the proper time difference "dt_0". So, in the Schwarzschild solutions, we need to consider two different *measurements* of time, namely "dt_0" and "dt_∞" and one *hypothetical but synchronized* time difference "dt". We need the Noether time difference "dt" to establish speed within the Noether frame as ds/dt, in which "ds" is the space distance. The space distance "ds" equals in both coordinate systems:

$ds^2 = dx^2 + dy^2 + dz^2 = dr^2 + r^2.d\theta^2 + r^2.\sin^2\theta.d\varphi^2$ [m^2] space distance "ds"

Here you begin to see the outlines of the Schwarzschild Solution. The proper time "dt_0" is measured on a local (proper) clock. Proper time is measured in proper seconds [s_0]. A space-time distance is thus measured in proper units meter [m_0], since radar equipment is used to establish distance. Einstein modified Minkowski's formula to describe *curved* space-time. Minkowski's original formula is in both polar and Cartesian coordinate systems:

$ds^2 = c^2.dt_0^2 = c^2.dt^2 - dz^2 - dy^2 - dx^2$ [m_0^2] Minkowski space-time (Cartesian)
$ds^2 = c^2.dt_0^2 = c^2.dt^2 - dr^2 - r^2.d\theta^2 - r^2.\sin^2\theta.d\varphi^2$ [m_0^2] Minkowski space-time (polar)
$ds^2 = c^2.dt_0^2 = c^2.dt^2 - ds^2$ [m_0^2] Minkowski space-time

We can now look at Einstein's change of mind about the determinant condition.

Speed of light

What we know is that the speed of light "c" locally must be invariant to changes in space. In other words: dx/dt, dy/dt, and dz/dt in vacuum must be equal to "c". We also know that at infinity for the Schwarzschild Solution (Newtonian limit), the solution is known: $g_{00} = 1$, $g_{11} = -1$, $g_{22} = -1$, and $g_{33} = -1$, then the Minkowski formula reappears since space is then Euclidean. What we also know is that the speed of light in the Schwarzschild Solution is different at different locations. Light has a proper time of zero, $dt_0 = 0$.

Looking at Schwarzschild Solution, we get $dx/dt_\infty = g_{00}.c$, $dy/dt_\infty = g_{00}.c$, and $dz/dt_\infty = -g_{00}.c / g_{11}$. Schwarzschild wrote (translated, see formula 44 of his original manuscript of February 1916): "The velocity of light inside our sphere becomes: $v = 2 / (3\cos\chi_a - \cos\chi)$, growing from $1/\cos\chi_a$ on the surface to $2 / (3\cos\chi_a - 1)$ at the core". *Schwarzschild confirms the variable local speed of light, in his second solution.*

Einstein did not replace the g = –1 demand for anything else. Einstein did not repair the Schwarzschild Solutions either. Einstein worked until his death on a better version of his field theory, but never published an update, apart from dropping the "cosmological constant" (now called "dark energy").

Eddington "dropped the suffix"

In 1917 J. Droste produced the first solution after Einstein's final publication. Later in 1922, Eddington described the Schwarzschild solution (to a mass-point) in clear English with comments. He copied Droste as did many authors (including Misner, Thorne, and Wheeler in "Gravitation") since then. However, Droste nor he got the same result as Karl Schwarzschild. They replaced Schwarzschild's auxiliary variable "R" by the coordinate "r":

$c^2.dt_0^2 = g_{00}.c^2.dt_\infty^2 + g_{11}.dr^2 + g_{22}.r^2.d\theta^2 + g_{33}.r^2.\sin^2\theta.d\varphi^2$ $[m_0^2]$ Droste/Eddington Solution

In this equation is "g_{00}" equal to $(1 - R_S / r)$, while $g_{11} = 1 / g_{00}$. This in contract with Karl's solution, in which g_{00}" equals $(1 - R_S / R)$ and $R^3 = r^3 + R_S^3$. Droste got an outcome in which g_{22} and g_{33} are *not equal to minus one*, the value Karl Schwarzschild arrived at based on $g = -1$.

The authors fully agree that Droste was on the right track. Fact is that current publications of the Schwarzschild Solution are still fumbling with statements to get to the *presumed* correct outcome with both g_{22} and g_{33} equal to minus one. Those are social arguments, would you dare to be a scientist? Would you dare to listen to Emmy Noether, even though she was ignored by Einstein, Eddington, Misner, Thorne, and Wheeler?

Noether's theorem restores historic errors

The authors of this article have worked out the replacement demand such that Noether's theorem and the Newtonian limit are adhered to. However, before we can do so, we need to look into the Shapiro Solution and experiment. Why? To compute the speed of light in gravitation (ds/dt), we do not need the time difference as measured from far "dt_∞", but the Noether time difference "dt". The Shapiro Solution and experiment are the key, this solution provides us with this needed relation.

Shapiro sent a radar signal to Venus and back in 1964, long after Einstein's death. The space around the sun is denser than far away from the sun. Because of the *added* space (not the *curvature* of space as pointed out by Shapiro), the radar signal is delayed in the Euclidean Noether frame ($dt_\infty > dt$). The Shapiro Solution gives us this relation in a single formula, the "Shapiro delay":

$dt_\infty = dt / (1 - R_S / r)^2$ [s] Shapiro Solution (delay)

This relation is confirmed by his experiment. *Note that the Schwarzschild Solution would have to come to the same outcome, but does not*. Because of the "dropping of the suffix" by Eddington the terms g_{22} and g_{33} became minus one. When the radar signal passes the sun, the largest change in coordinates is "dθ" if we put the polar coordinate system such that "φ" is zero and the origin is located in the center-of-mass of the sun.

Close to the sun "dr" equals zero, while "dt_0" is by definition zero for light. The equation would then become: $g_{00}.c^2.dt_\infty^2 = -g_{22}.r^2.d\theta^2 = -g_{22}.c^2.dt^2$, since $r.d\theta = c.dt$. With $g_{22} = -1$ (Droste/Eddington) and $g_{00} = 1 - R_S / r$, we would get: $dt_\infty = dt / (1 - R_S / r)$, *which is missing the square in the denominator*, see the Shapiro Solution above.

Covariant metric tensor of Schwarzschild's Solution

If Eddington would not have "dropped the suffix" and had seen that $g_{11} = g_{22} = g_{33}$ (Noether's theorem), he could have predicted the Shapiro delay. The Shapiro solution (delay) changes the *generic Schwarzschild Solution* into functions of the Noether frame (t,x,y,z) or (t,r,θ,φ) of:

$c^2.dt_0^2 = g_{00}.c^2.dt^2 + g_{11}.dr^2 + g_{22}.r^2.d\theta^2 + g_{33}\, r^2.\sin^2\theta.d\varphi^2$	$[m_0^2]$	*Repaired* Solution (polar)
$c^2.dt_0^2 = g_{00}.c^2.dt^2 + g_{11}.dz^2 + g_{22}.dy^2 + g_{33}.dx^2$	$[m_0^2]$	*Repaired* Solution (Cartesian)

Note that "g_{00}" is a different one than in the previous equations, because of the use of "dt" instead of "dt_∞". In the repaired solution is $g_{00} = -g_{11} = 1 / (1 - R_S / r)$. Now we can finally apply Noether's theorem, demanding that the constants of nature are invariant to location in order to prove momentum conservation. In other words: dx/dt = dy/dt = dz/dt = c for light in vacuum. This means for the Repaired Solution: $g_{00} = -g_{11} = -g_{22} = -g_{33} = 1 / (1 - R_S / r)$.

Repaired Schwarzschild Solution

We thus get a much simplified Schwarzschild solution in either Cartesian or polar coordinates, which *satisfies Einstein's relativity principle as well Noether's theorem*:

$c^2.dt_0^2 = (c^2.dt^2 - ds^2) / (1 - R_S / r)$	$[m_0^2]$	*Repaired* Schwarzschild Solution

With ds = v.dt for any mass-particle in the gravitational field within a Noether frame, we get:

$dt_0 = dt / \sigma.\gamma = \sigma.dt_\infty / \gamma$	$[s_0]$	*Repaired* Schwarzschild Solution
$\sigma = (1 - R_S / r)^{½}$	[ ]	gravitation-factor
$\gamma = (1 - v^2 / c^2)^{-½}$	[ ]	boost-factor

Note how Special Relativity (SR) reappears when the *gravitation-factor* "σ" equals one. The most beautiful outcome of the Repaired Solution is the *extended SR formula*:

$E = m.c^2 = \sigma.\gamma.m_0.c^2$	[J]	Energy of mass-particle

Based on continuity, we can also provide you with the solution for the inside of a sphere with radius "R". Do not confuse Schwarzschild's discarded auxiliary variable "R" with the radius "R" of the sphere of incompressible liquid:

$dt_0 = dt / \sigma.\gamma = \sigma.dt_\infty / \gamma$	$[s_0]$		Repaired Schwarzschild Solution
$\sigma = (1 - R_S / r)^{½}$	[ ]	$r \geq R$	gravitation-factor (in vacuum)
$\sigma = (1 - 3G.M / c^2.R + G.M.r^2 / c^2.R^3)^{½}$	[ ]	$r \leq R$	gravitation-factor (inside sphere)
$\gamma = (1 - v^2 / c^2)^{-½}$	[ ]		boost-factor (Special Relativity)

With the *Repaired Schwarzschild Solution*, we can explain the Shapiro delay, compute the escape speed from the earth, compute the core temperature of the earth and sun, and reject black hole "singularities", since $R_S < R$, in agreement with the original Schwarzschild Solution! From a scientific point of view, Einstein and Noether are united

Let us, based on this outcome, redefine Einstein's g = –1 condition.

Covariant metric tensor of Schwarzschild's Solution

Covariant metric tensor requirements

The covariant metric tensor "g_{ii}" and the coordinate system chosen must abide by the following conditions in order to fulfill Noether's theorem:

1. At infinity, the Minkowski formula must reappear (Newtonian limit),
2. All constants of nature are invariant to location and time within a Noether frame,
3. All 16 elements of "g_{ii}" do not have a unit of measurement,
4. Coordinate transformations must come to the same measurable outcome "dt_0"

By these demands, Einstein's principles are still valid. The covariant metric tensor of the *repaired* Schwarzschild Solution in terms of the measurable time difference "dt_∞" is found in figure 2:

$$c^2.dt_0^2 = \begin{bmatrix} c.dt_\infty & dr & d\alpha & d\beta \end{bmatrix} \begin{bmatrix} \sigma^2 & 0 & 0 & 0 \\ 0 & -\sigma^{-2} & 0 & 0 \\ 0 & 0 & -\sigma^{-2} & 0 \\ 0 & 0 & 0 & -\sigma^{-2} \end{bmatrix} \begin{bmatrix} c.dt_\infty \\ dr \\ d\alpha \\ d\beta \end{bmatrix}$$

$$\sigma^2 = 1 - R_S / r \qquad g_{ii} = -1 / g_{00} \quad i = 1,2,3 \qquad d\alpha = r.d\vartheta \qquad d\beta = r.\sin\vartheta.d\varphi$$

$$c^2.dt_0^2 = \begin{bmatrix} c.dt_\infty & dz & dy & dx \end{bmatrix} \begin{bmatrix} \sigma^2 & 0 & 0 & 0 \\ 0 & -\sigma^{-2} & 0 & 0 \\ 0 & 0 & -\sigma^{-2} & 0 \\ 0 & 0 & 0 & -\sigma^{-2} \end{bmatrix} \begin{bmatrix} c.dt_\infty \\ dz \\ dy \\ dx \end{bmatrix}$$

$$\sigma^2 = 1 - R_S / r \qquad g_{00} = \sigma^2$$

Figure 2: covariant metric tensor g of the *repaired* Schwarzschild Solution

The following coordinate transformations and coordinate *difference* transformations:

$z' = r.\cos\theta$ $dz = dr$

$y' = r.\sin\theta.\sin\varphi$ $dy = r.d\theta$ $d\alpha = dy$

$x' = r.\sin\theta.\cos\varphi$ $dx = r.\sin\theta.d\varphi$ $d\beta = dx$

ensure a dimensionless covariant metric tensor. At $r = \infty$, $\sigma = 1$, a Euclidean space and the Minkowski formula reappears (Newtonian limit). The Cartesian (dx,dy,dz) and (x',y',z') have a different origin. The Cartesian (dx,dy,dz) origin is located at (x',y',z') = (r,θ,φ). The volume scalar (Einstein's demand) ensures the same measurable outcome and comes to:

$V = c.dt.dx.dy.dz = c.dt.dr.d\alpha.d\beta = c.dt.r^2.dr.d\theta.\sin\theta.d\varphi$ [m^4]   4D volume scalar

When you now compare figure 2 to figure 1, you see how the Shapiro solution fits in! Einstein's determinant condition $g = -1$ is replaced by $g_{ii} = -1 / g_{00}$ for i = 1,2,3, using "dt_∞" as measured time difference instead of the immeasurable Noether time difference "dt".

Covariant metric tensor of Schwarzschild's Solution

Book II: Repairing Schwarzschild's Solution

As stated before, with the *Repaired Schwarzschild Solution*, we can explain the Shapiro delay, compute the escape speed from the earth, compute the core temperature of the earth and sun, and reject black hole "singularities", since $R_S < R$, in agreement with the original Schwarzschild Solution! Details can be found in our book II called: "Repairing Schwarzschild's Solution".

The escape speed from Earth varies from 11,180 to 11,200 [$m.s^{-1}$], compared to the escape speed as specified by NASA of 11,190 [$m.s^{-1}$], see paragraph 5.03. The temperature of the core of the earth must be at least 5,380 [K], see paragraph 6.12 and the core temperature of the sun must be at least 15,500,000 [K] based on the repaired Schwarzschild Solution, see paragraph 6.13.

The radius "R" of a static sphere of incompressible liquid is at least $1½R_S$, which is even further away than the factor $9/8R_S$ of Karl Schwarzschild and Misner, Thorne, and Wheeler in "Gravitation". This means that there is no "event horizon" of a static black hole. Thorne's tortoise coordinates cannot be applied, a static black hole cannot collapse to within its Schwarzschild radius. The extremely high temperature at the core of a *static* black hole will limit the mass to a maximum of 2.3 sun masses. However, all black holes rotate, just like all neutron stars rotate.

Because of high rotational speeds, massive black holes take the shape of an oblate spheroid with jets (at relatively low rotational speeds) or the shape of a torus (at high rotational speeds), see chapter 9.

More information?

Our three books (www.loop-doctor.nl) describe the repair of Einstein's Relativity for Noether's theorem in full detail. We hope you get as many "aha" experiences as we did,

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