Einstein debates with Noether (part 1)

Emmy Noether and Albert Einstein

Emmy Noether, born in 1882, studied at the University of Göttingen in 1903 and 1904, attending lectures given by Karl Schwarzschild, Hermann Minkowski, and David Hilbert. Emmy worked at the University of Göttingen from 1915 to 1933 as lecturing assistant to David Hilbert.

Albert Einstein, born in 1879, moved to Berlin in 1914 to work at the Prussian Academy of Sciences in Berlin. Göttingen and Berlin are more than 300 [km] away from each other, so most correspondence with Emmy went by mail. Some of those letters have been saved, but the main issue is the *conflict* between Einstein's theory of General Relativity as published at the end of 1916 in "die Annalen der Physik" 49 and Noether's theorem published in 1919.

Emmy's criticism about Albert's theory is found in 1918 in her thesis "Invariante Variationsprobleme" (translated by Tavel[1]): "Hilbert enunciates his assertion to the effect that the failure of proper laws of conservation of energy is a characteristic feature of the general theory of relativity". Although written by Emmy, she could not be a professor as a woman, so the correspondence was officially done by professor Hilbert.

The following conversation is *fictitious*, unless written between double quotes (and translated), but are based on Noether's theorem and Einstein's theory of General Relativity. The date was Tuesday April 4, 1922, when Albert was 43 and Emmy 40 years old.

Question Emmy: Why did you introduce a variable speed of light in your general theory?
Albert: Gravitation must have an influence on the speed of light "c"; especially away from the origin. "The principle of the constancy of the speed of light holds good according to this theory in a different form from that which usually underlies the ordinary theory of relativity": $c = c_0.(1 + \Phi / c^2)$, in which "Φ" is the gravitational potential and "c_0" the invariant speed of light in the origin".[2]

Question Emmy: If you accept a *variable* speed of light "c" in gravitation, how could you then unify the speed of light (c = 1 light-second per second)?
Albert: I wanted to get rid of the constants of nature: speed of light "c" and Newton constant "G"; I needed *units meter only* in tensor analysis. Besides that, the deviation of the speed of light "c" from its constant value "c_0" is in normal practice so small that it is *immeasurable*.

Question Emmy: So your general theory is not meant for very strong gravitation?
Albert: That is correct, general relativity gives better results than Newton's laws, but is a first order approximation only. The gravitational field of the sun and planets are well described by my theory; think of the orbit of Mercury and the bending of starlight around the sun at an eclipse of the sun.

Question Emmy: What do you think of Karl Schwarzschild's *exact* solutions then?
Albert: As said before, my theory is not meant for strong gravitation, we then have to include the variable speed of light again as a function of the strength of the gravitational field. However, I find it "not done" to criticize users of my own theory, which is hard enough as it is.

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[1] Tavel. M. "Invariant Variation Problems", Transport Theory and Statistical Physics 1971, 1(3) p. 186 - 207.
[2] Einstein A. "Über den Einfluss der Schwerkraft auf die Ausbreitung des Lichtes" Annalen der Physik 35, 1911

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Question Emmy: Why did you drop your determinant condition g = –1 after Karl's death in 1916?
Albert: I'm so sorry that Karl deceased at such a young age. Karl made use of this demand to get to his *exact* solutions. However, his solutions made it clear to me that this g = –1 demand does not fulfill its intended *purpose*. Besides, your theorem is also in conflict with g = –1 (Emmy nods).

Question Emmy: Which purpose?
Albert: I set out to ensure that coordinate (system) transformations are such that the measured proper time difference "*ds*" (= c.dt_0) remains "ascertainable by rod-clock measurements of point events infinitely proximate in space-time".[3]

Question Emmy: After you dropped the g = –1 demand, a Dutch mathematicians (Droste) has solved your equations for a mass-point in 1917. He began with an incomprehensible sentence "..,we are at liberty to choose instead of r a new variable which will be such a variable of r, that in ds^2 the coefficient of the square of its differential becomes unity"[4]. Did he work towards your g = –1?
Albert: I must admit that dropping the g = –1 demand was a last minute addition and is only written in a *footnote at the end of paragraph 19*. He might not have read it.

Question Emmy: Eddington "dropped the suffix"[5] four years later to get to g = –1, did you discuss that with him?
Albert (annoyed): Eddington made me famous after I was ignored for fifteen years! I was not going to criticize him on a detail of my own making which does not make much difference within our solar system.

Question Albert (irritated): Why do you ask all these questions, I feel drilled!
Emmy (apologetic): Sorry Albert, I did not mean it as drilling, I merely wanted to establish the facts before suggesting a few modifications to your brilliant theory.

Question Albert (still irritated): What modifications, it took me ten years to get this far?
Emmy: we need to work on a constant speed of light away from the origin and to ensure your initial demand remains "ascertainable by rod-clock measurements of point events infinitely proximate in space-time".

Question Albert (suddenly interested): "you are a creative mathematical genius"[6], would you be using your theorem?
Emmy (relieved): Of course, that is what I do. Your principles and my theorem can create a whole new theory!

Question Albert: So what would you do?
Emmy: Let us begin with defining a reference frame in which we can guarantee energy and momentum conservation. Such a reference frame must be everywhere the same (homogenous) and the same in all directions (isotropic), while the time differences are limited such that the laws of

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[3] Einstein A. "The General Theory", Annalen der Physik, 49, 1916 paragraph 4 formula (3) and following.
[4] Droste J. KNAW proceedings 19 I ,1917 page 199.
[5] Eddington A. "The Mathematical Theory of Relativity", 1921 after formula (38.13).
[6] Einstein A. at Noether's funeral in 1935

nature and its constants like the speed of light "c" remain the same. Such a reference frame is Euclidean or "flat" and has a synchronized time "t" throughout.
Question Albert (feeling guilty): We will call such a reference frame a "Noether frame". However, I see a problem, my space-time is curved, not Euclidean, how will you deal with that?
Emmy: Thank you for naming me, the men around you would rather not see a woman in science! To answer your question: *space-time curves within a hypothetical Noether frame*, we call this curvature "extrinsic curvature", *the fabric of space-time itself*!

Question Albert: Brilliant Emmy, but how do you cope with real time difference measurements within a *hypothetical* Noether frame?
Emmy: Unfortunately, we end up with *three* different time differences: "*ds*" (= c.dt_0) as measured on a clock of the proper observer at speed and in gravitation, "dt_∞" as measured from far as defined by Karl Schwarzschild, and a new *hypothetical* time "dt" within a Noether frame.

Question Albert: I know the relation between proper time difference "dt_0" and "dt_∞", that is my defined gravitational potential: "Φ"[7]: $dt_0^2 = \Phi.dt_\infty^2$ (see appendix). However, what is the relation between the needed "dt" in the Noether frame and the other two measured time differences?
Emmy: Well, someday a brilliant professor will *experimentally* prove the required highly accurate relation between your relativity and my theorem. Let us call this professor SHAPIRO (Such a Highly Accurate Prove of the Interdependence of your Relativity and my Official theorem).

Question Albert: So how does that relate to your theorem and the constancy of the speed of light?
Emmy: We need to prove that dx/dt = dy/dt = dz/dt = c for light in vacuum in gravitation in which "dt_0" is zero. Currently we only have "dt_∞". So, if $dt_\infty = dt / \Phi$ is indeed true, and it must be in the logic of physics, we ensure the constancy of the speed of light!

Question Albert (enthusiastic): We then get the Schwarzschild Solution to a sphere of incompressible liquid within a Noether frame, which is much simpler: $dt_0 = dt / \sigma.\gamma$ (see appendix)! This solution unites the Schwarzschild Solution with my theory of Special Relativity! That looks very promising to me Emmy, but how do we generalize this to my theory of General Relativity?
Emmy: Replace your g = –1 demand by the demand that the proper time measurement "dt_0" must remain the same after coordinate (system) transformations, as you stated in paragraph 4!

Albert: Dear Emmy, We are such a great team together, only now I realize to my disgrace how I have ignored you, I am so sorry.
Emmy: No problem Albert, I always have kept faith that recognition would come.

Albert: Emmy, shall we meet in here in Göttingen again next week to discuss the *inside* of the sphere of incompressible liquid in order to complete the solution?
Emmy: I would love to; to see the pieces of the puzzle of physics coming together is wonderful.

Note of the authors: Emmy Noether is always praised verbally, but her theorem is not often included in the books about relativity. There is not even a single reference to her or her theorem in the book "Gravitation" of 1975 (1279 pages about relativity) written by Misner, Thorne, and Wheeler. Emmy Noether, the *ignored* scientist, see www.loop-doctor.nl.

[7] Einstein A. "The General Theory", Annalen der Physik, 49, 1916 paragraph 21, formula (67) onwards.

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Appendix for experts

Time-like Schwarzschild solution to a sphere of incompressible liquid:

$ds^2 = c^2.dt_0{}^2 = \Phi.c^2.dt_\infty{}^2$	$[m_0{}^2]$	time-like Schwarzschild Solution	(1)
$\Phi \;= \sigma^2 = (1 - R_S / r) \qquad r \geq R$	[ ]	Einstein's gravitational potential "Φ"	(2)
$R_S \;= 2G.M / c^2$	[m]	Schwarzschild radius "R_S"	(3)

In these equations is "R" the radius of the sphere, "σ" the square root of the gravitational potential "Φ", is "dt_0" the proper time difference as measured by a proper observer of a point event infinitely proximate in space-time, and is "dt_∞" the same time difference as measured on a (cesium) clock far away from the sphere. The Schwarzschild radius of the earth is 0.008 87 [m] and of the sun 2,953 [m].

The *real* Shapiro Solution and experiment of 1964 (Shapiro delay) is based on:

$dt_\infty = dt / \Phi$	[s]	Shapiro Solution	(4)

Combining formula (1) and (4), gets us:

$ds^2 = c^2.dt_0{}^2 = c^2.dt^2 / \Phi$	$[m_0{}^2]$	time-like Solution (Noether frame)	(5)

Expanding the time-like solution to a full solution based on a constant speed of light "c" within the Noether frame:

$c^2.dt_0{}^2 = (c^2.dt^2 - dz^2 - dy^2 - dx^2) / \Phi$	$[m_0{}^2]$	Solution (Cartesian coordinates)	(6)
$c^2.dt_0{}^2 = (c^2.dt^2 - dr^2 - r^2.d\theta^2 - r^2.\sin^2\theta.d\varphi^2) / \Phi$	$[m_0{}^2]$	Solution (polar coordinates)	(7)
$ds^2 = dx^2 + dy^2 + dz^2$	$[m^2]$	distance "ds" in Noether frame	
$ds^2 = dr^2 + r^2.d\theta^2 + r^2.\sin^2\theta.d\varphi^2$	$[m^2]$	distance "ds" in Noether frame	
$c^2.dt_0{}^2 = (c^2.dt^2 - ds^2) / \Phi$	$[m_0{}^2]$	Solution (sphere)	(8)

Note that formula (8) is a solution *independent of the coordinate system chosen*, exactly according to Einstein's "relativity principle". A particle moves distance "ds" (not italic) with speed "v" in time "dt" within the Noether frame: ds = v.dt. Substituting ds = v.dt in formula (8) results in:

$c^2.dt_0{}^2 = (c^2.dt^2 - v^2.dt^2) / \Phi$	$[m_0{}^2]$		
$dt_0{}^2 = (1 - v^2 / c^2).dt^2 / \Phi$	$[m_0{}^2]$		
$\gamma \;= (1 - v^2 / c^2)^{-1/2}$	[ ]	boost-factor "γ"	(9)

We thus get the *Repaired* Schwarzschild Solution of a sphere of incompressible liquid:

$dt_0 \;= dt / \sigma.\gamma = \sigma.dt_\infty / \gamma$	$[s_0]$	Solution (sphere)	(10)