

Albert Einstein's space-time and the speed of light

Introduction and relevance

Albert Einstein (1879 - 1955) was a pioneer of space-time. In 1905, he published his theory of Special Relativity, in which he discovered the energy in mass ($E = m.c^2$) based on the *constancy* of the speed of light to all observers. He also introduced time dilation (time going slower at high speed), based on the Lorentz transformation in the origin of the transformed frame.

In 1916 he published his theory of General Relativity, in which he explained the curious orbit of Mercury (perihelion precession) and predicted the bending of starlight around the sun at a solar Eclipse (1.75 arc-seconds, measured by Eddington). He replaced the Lorentz transformation by the Minkowski formula (differential and equal in all directions) which is principally different from the Lorentz transformation and applicable in both theories.

Time at speed

If humanity will ever be capable of travelling very close to the speed of light, amazing things would happen to you as high-speed traveler. Because your clock (and thus time) ticks so much slower than on earth, gigantic distances could be travelled in very little time. A trip to the nearest exoplanet and back would take you less than a year. However, upon return, most of your family and friends have died and your grandchildren have come of age. That is time dilation in its ultimate form. Time dilation goes together with “distance contraction”; to you as traveler at high-speed, the distances seem to get very small. However, when your speed diminishes, the distances become longer again!

Space-time in Special relativity

Einstein's tutor Minkowski replaced the Lorentz transformation by a differential formula in which the speed of light is constant, but is not limited to a constant speed of the proper traveler. Let us talk you through this all-important formula of Minkowski, the essence of space-time:

$$c^2.d\tau^2 = c^2.dt^2 - dx^2 - dy^2 - dz^2 \quad [m^2] \quad \text{Minkowski formula (Cartesian coordinates)} \quad (1)$$

$$c^2.d\tau^2 = c^2.dt^2 - ds^2 \quad [m^2] \quad \text{Minkowski formula (space distance ds)} \quad (2)$$

In this formula is “c” the speed of light of 299.792.458 [m.s⁻¹], “ $d\tau$ ” the proper time difference as measured on the clock of the travelling observer (the proper observer) in proper seconds [s_0], “dt” the time difference in [s] within the reference frame in which the proper observer is travelling, and are “dx”, “dy”, and “dz” the changes in location of the proper observer within the reference frame in [m]. Units are placed between [] and proper variables are written in *italic* to indicate their variability.

The outcome is the product of the speed of light “c” times the proper time “ $d\tau$ ” (squared). The term $c.d\tau$ is also referred to as the line element “ ds ” ($ds = c.d\tau$). The validity of the Minkowski formula is depending on a Euclidean (straight or flat) reference frames with synchronized clocks.

For example, when a proper observer with clock travels at 60% of the speed of light in the “x” direction such that $ds = dx = 0.6c$ [m] and $dt = 1.0$ [s], then the proper observer will measure a proper time difference “ $d\tau$ ” of 0.8 [s_0] over that same distance of “dx” within the reference frame. The proper observer notices that its clock ticks slower by 20%. This phenomena is called “time dilation”, time slowing down when a proper observer travels fast within a reference frame.

Albert Einstein's space-time and the speed of light

That reference frame could be the reference frame of the earth. The slowing down of the clock and time of the proper observer is confirmed by the lifetime of muons. Muons are created by fast particles of the solar wind colliding with our atmosphere at about 12 [km] height. These muons only live 2.2 proper microseconds [s_0] on average. The collisions cause the muons to speed towards the surface of the earth with nearly the speed of light. If there would be no time dilation, these muons would not reach the surface of the earth; in 2.2 microseconds only about 660 [m] can be reached.

However, most of these muons reach the surface of the earth, proving that the clocks of the proper observers of the muons tick at least 20 times as slow as our clocks on earth. In summary, high-speed proper observers of mass-particles have slower clocks and thus measure less time than the observers on earth.

Another example of time dilation is light in vacuum. Light goes with the speed of light “c” within the reference frame. If the light goes in the “x” direction, you get $dx = c \cdot dt$ [m]. The Minkowski formula then results in a proper time of zero ($dt_0 = 0$). This means that the clock and the time of the proper observer stand still. This is the ultimate time dilation. The amount of slowing down is called the boost-factor “ γ ” (Greek gamma):

$$\gamma = (1 - v^2 / c^2)^{-1/2} \quad [] \quad \text{boost-factor} \quad (3)$$

In this formula is “v” the speed of the proper observer and is the factor “ $-1/2$ ” an easy way of describing one divided by the square root in printable language. Since $ds = v \cdot dt$, we get the formula of time dilation by combining formula (2) and (3):

$$dt_0 = dt / \gamma \quad [s_0] \quad \text{time dilation of proper observer} \quad (4)$$

The boost-factor “ γ ” ranges from 1.0 ($v = 0$) to infinity ($v = c$).

Space-time in General Relativity

Einstein used the Minkowski formula in his theory of General Relativity as foundation, but with a twist. He *abandoned* the constancy of the speed of light, which he introduced in his theory of Special Relativity!

To understand that, let us look at the simplest solution to his theory, the Shapiro experiment and solution. Shapiro sent a radar signal to Venus and back, while Venus is nearly at the other side of the sun. This radar signal is delayed because of the gravitational field of the sun (the so-called Shapiro delay). The essential differential formula is:

$$c^2 \cdot dt_0^2 = (1 - R_s / r) \cdot c^2 \cdot dt^2 - dx^2 / (1 - R_s / r) \quad [m_0^2] \quad \text{Shapiro Solution} \quad (5)$$

$$R_s = 2G \cdot M / c^2 \quad [m] \quad \text{Schwarzschild radius of the sun} \quad (6)$$

In these formulas is “ R_s ” the Schwarzschild radius (= 2,953 meter), is “G” the Newton constant, is “M” the mass of the sun, and is “r” the distance to center-of-mass of the sun. We know that light travels with the speed of light in vacuum with a proper time of zero ($dt_0 = 0$).

Albert Einstein's space-time and the speed of light

We can thus deduce the (mistaken) speed of light in the Shapiro Solution as:

$$v = dx / dt = (1 - R_s / r)^2 \cdot c \quad [\text{m.s}^{-1}] \quad (\text{mistaken) speed of light in Shapiro Solution} \quad (7)$$

The term $(1 - R_s / r)^2$ is close to one, so the speed of light “v” would still be very close to “c”; that is, *if* the speed of light would be variable in vacuum. *If* the speed of light would be somewhat lower than “c”, you would get an accumulated delay compared to the situation in which the sun would not be there, the Shapiro delay. However, all experiments indicate a constant speed of light in vacuum! How do we deal with this conflict?

Here comes the issue with General Relativity: If the speed of light is *not constant* (in conflict with Special Relativity), how can Einstein then normalize the speed of light to one ($c = 1$) as he did in his theory of General Relativity? Normalization is based on a constant, independent of distance “r”. However, you can still see the dependence of “r” in formula (7). In fact, it is not allowed in physics or mathematics to normalize a *variable* speed of light to one!

An additional problem to the variability of the speed of light are the experiments. All experiments in the gravitational field of the earth indicate a constant speed of light “c”, even in the latest versions of the Michelson-Morley experiment. The gravitational field of the earth does not influence the speed of light in any measurable way. Yet another additional problem to the variability of the speed of light, is that the SI organization has defined the unit meter as the distance light travels in vacuum in $1 / c$ seconds, assuming (correctly) a constant speed of light under all circumstances.

It gets even worse, the abandonment of the constancy of the speed of light is in conflict with Noether's theorem. *Energy conservation cannot be proven for a variable speed of light*. Emmy Noether warned Albert Einstein about this in 1918, but Einstein ignored her, Noether the *ignored* scientist. The authors support Noether's theorem, and thus reject Einstein's *variable* speed of light in his theory of General Relativity. To explain the Shapiro experiment with a constant speed of light, we must realize that clocks tick slower the stronger the gravitation. The only place where the gravitation does not influence the clock is at infinity. So, the “dt” of formula (5) must be read as measured from infinity: “ dt_∞ ”.

When doing so, we get a relation between the time difference as measured from far “ dt_∞ ” and the time difference as it would be if no gravitation was around “dt”: $dt_\infty = dt / (1 - R_s / r)$. This relation lies at the heart of the repair of the Schwarzschild Solution, see our book II: “Repairing Schwarzschild's Solution”. Our three books (www.loop-doctor.nl) describe the repair of Einstein's Relativity for Noether's theorem in full detail. We hope you get as many “aha” experiences as we did,

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