Earth's core temperature *calculated*, not *guessed*

Noether's law of energy conservation explains high core temperature

Introduction and relevance

The temperature of the inner core of the earth is deduced to be around 5,700 [K] based on the melted iron-nickel creating a magnetic field and the reflected waves caused by earthquakes. The gradual cooling of the Earth's interior is thought to be about 100 degrees [K] per billion years. The cause of this high core temperature is currently thought to be radioactive decay (80%) and residual heat (20%), see https://en.wikipedia.org/wiki/Earth reference [110].

The authors provide a different explanation (than the 80% caused by radioactive decay) to the high temperature: energy conservation. The authors demonstrate that the main cause of the high temperature of the inner core is the constancy of energy within a massive body in thermal equilibrium according to Noether's conservation laws. In a first computation, the temperature at which the inner core will stabilize, equals 5,380 [K]. The higher temperature of 5,700 [K] as currently estimated, could still be explained by some residual heat and some radioactive decay. In other words, the authors argue that it is *not the radioactive decay, but energy conservation, which is mainly responsible for the high temperature at the core of the earth.*

On top of that, the authors claim that the *very high temperature at the core of the sun is not caused by the fusion process, but reversely, that the fusion process within the sun is caused by the high temperature*. The high temperature of the core of the sun can be computed in the same way as the computation of earth's core, resulting in a core temperature of the sun of 15,550,000 [K], irrespective of the fusion process!

Current vision on core temperature (Wikipedia)

"Earth's internal heat comes from a combination of residual heat from planetary accretion (about 20%) and heat produced through radioactive decay (80%). The major heat-producing isotopes within Earth are potassium-40, uranium-238, uranium-235, and thorium-232. At the center, the temperature may be up to 6,000 °C, and the pressure could reach 360 GPa. Because much of the heat is provided by radioactive decay, scientists postulate that early in Earth's history, before isotopes with short half-lives had been depleted, Earth's heat production would have been much higher. This extra heat production, twice present-day at approximately 3 GPa, would have increased temperature gradients with radius, increasing the rates of mantle convection and plate tectonics, and allowing the production of uncommon igneous rocks such as komatiites that are rarely formed today.", see https://en.wikipedia.org/wiki/Earth and:

"The temperature of the inner core can be estimated by considering both the theoretical and the experimentally demonstrated constraints on the melting temperature of impure iron at the pressure at the boundary, which iron is under, of the inner core (about 330 GPa). These considerations suggest that its temperature is about 5,700 K (5,400 °C; 9,800 °F). The pressure in the Earth's inner core is slightly higher than it is at the boundary between the outer and inner cores: it ranges from about 330 to 360 gigapascal (3,300,000 to 3,600,000 atm). Iron can be solid at such high temperatures only because its melting temperature increases dramatically at pressures of that magnitude (see the Clausius–Clapeyron relation)", see https://en.wikipedia.org/wiki/Inner_core.

Earth's core temperature *calculated*, not *guessed*

Noether's law of energy conservation explains high core temperature

The authors calculate the inner core temperature based on energy conservation; the sum of (high) potential energy "E_{pot}" and (low) kinetic energy "E_{kin}" at the surface equals the sum of the zero gravitational potential and (high) kinetic energy at the core of the earth:

$$E = E_{pot} + E_{kin} = \text{constant} \qquad [J] \qquad (0)$$

To understand this , let us drill an imaginary tunnel through the earth.

A tunnel through the earth

Imagine a tunnel from your location, through the center-of-mass, to the other side of the earth. Imagine this tunnel to be vacuumed and frictionless. What would happen to a falling pebble of mass "m"? The pebble would gain speed, get its highest speed at the center-of-mass and pop up at the other side of the earth at low speed (theoretically at zero speed).

Speed of mass-particles and temperature are related to each other by kinetic energy. The higher the speed "v", the higher the kinetic energy ($½m.v^2$), the higher the temperature. The pebble in the tunnel teaches us, purely based on energy conservation, that the temperature of the earth must be highest at the core. However, we must make a number of steps to be able to compute the temperature based on a number of basic assumptions.

The steps we must take are 1) establish the relation between average speed of particles and temperature, 2) establish the gravitational potential *within* the earth, and 3) establish the core temperature. This is a rather long and complicated computation involving the Schwarzschild solution to a sphere of incompressible liquid, Einstein's gravitational potential in his theory of General Relativity, using the hydrogen molecule as standard, and Noether's law of energy conservation. If you are interested in the analysis, see www.loop-doctor.nl/book-ii.

The earth is not a sphere of incompressible liquid, which would mean a constant density throughout the inner earth. However, there is no solution to Einstein's theory of General Relativity to a sphere of varying density, Einstein's gravitational potential *requires* a constant density throughout the sphere. In other words, the computations are based on a model of the earth with a constant density throughout. This model has worked well for the influence of the earth's gravitation on cesium clocks, so we expect this model to work well for the computation of the core temperature as well.

To get started, we will provide you with a Newtonian analysis based on the conservation of energy in the form of potential energy and kinetic energy within a sphere of incompressible liquid.

Newtonian computation

At the heart of the Newtonian computation two formulas are important; the first is (see https://en.wikipedia.org/wiki/temperature):

$E_{kin} = ½m.v^2 = 1½K_B.T_{kin}$ [J] average kinetic energy (1)

In this equation is "E_{kin}" the average kinetic energy of hydrogen molecules in [J], "m" the mass of a single hydrogen molecule (H_2) in [kg], "v^2" the average speed squared of the hydrogen molecules in [$m^2.s^{-2}$], "K_B" the Boltzmann constant (1.38 x 10^{-23}) in [$J.K^{-1}$], and "T_{kin}" the temperature based on kinetic energy of hydrogen molecules in [K].

Note that we have chosen the hydrogen molecule (H_2) to compute the kinetic energy. This is based on the assumption that temperature is independent of the molecule chosen, heavier molecules have lower speeds, but the kinetic energy remains the same. The authors have standardized on hydrogen as molecule to relate kinetic energy to speed, this is of importance to high temperatures when relativistic speeds are reached.

Also note that the temperature is based on kinetic energy in this formula, the alternative is the momentum "p" times speed "v" based temperature "T_{mom}" ($3K_B.T_{mom} = p.v$), which can be used for ideal gasses, photons, and extremely high temperatures. The earth being a liquid/solid of non-relativistic temperatures makes formula (1) the logical choice for Newtonian mechanics.

The second formula relates to potential energy, which is based on a sphere of incompressible liquid. It is based on the difference in the square roots of Einstein's gravitational potential "Φ_{in}" *within* a sphere of incompressible liquid, see www.loop-doctor.nl/book-ii:

$E_{pot} = ½G.M.m.r^2 / R^3$ $r \leq R$ [J] potential energy within a sphere (2)

In this equation is "E_{pot}" the potential energy within a sphere of incompressible liquid, "G" the Newton constant (6.67 x 10^{-23}) in [$N.m^2.kg^{-2}$], "M" the mass of the earth in [kg], "R" the radius of the earth (6,371,000 m), and "r" the distance from the center-of-mass of the earth in [m]. Note that the potential energy of a mass-particle like a pebble is zero at the center-of-mass (r = 0).

Also note that the potential energy above the surface of the earth (r > R) is very different, this is the familiar formula ("g" = 9.81 and "h" is the height above the surface):

$E_{pot} = m.g.h$ $r > R$ [J] potential energy above the earth (2a)

Both formulas (2) and (2a) are approximations which are only valid for small values of the Schwarzschild radius ($R_S = 2G.M / c^2$) relative to the real radius "R", for the explanation see www.loop-doctor.nl/book-ii. The earth has a radius of about 6,371 [km], while the Schwarzschild radius is about 8.88 [mm], which means that the approximations of formula (2) and (2a) are valid.

Computation of core temperature and speed

For the mass of the earth "M" of 5.97 x 10^{24} [kg], the hydrogen mass "m" of 3.37 x 10^{-27} [kg], we get the following energy levels at the surface (r = R) of the earth and at the core (r = 0):

Surface energy:

E_{pot}	$= ½G.M.m / R$	= 1.055 x 10^{-19}	[J]	potential energy at surface (r = R)
E_{kin}	$= ½m.v^2 = 1½K_B.T_{kin}$	= 0.059 x 10^{-19}	[J]	kinetic energy (T_{kin} = 283 Kelvin)
E_{surf}		= 1.114 x 10^{-19}	[J]	total surface energy

As mentioned before, see formula (0), energy conservation means that the earth-surface energy equals the earth-core energy: $E_{core} = E_{surf}$ = 1.114 x 10^{-19} [J]. $E_{core} = E_{pot} + E_{kin}$. E_{pot} at the core equals zero, see formula (2). So now we can derive the core temperature T_{kin} as follows:

$E_{kin} = ½m.v^2 = 1½K_B.T_{kin}$ = 1.114 x 10^{-19} [J] kinetic energy at the core

The Boltzmann constant "K_B" equals 1.38 x 10^{-23} in [$J.K^{-1}$], which results in T_{kin} = 5.380 [K]. Note that the sum of kinetic and potential energy "E" remains the same 1.114 x 10^{-19} [J] all the way. This illustrates the high core temperature of 5,380 [K] at the core of the earth *without any radioactive decay*. Of course, the radioactive decay and the residual heat do have an influence as long as the earth is not in a thermal equilibrium, see next paragraph.

If you want to know the speed of the pebble at the core, we can combine (1) and (2), while r = R and the kinetic energy of the pebble at the surface is zero:

$½m.v^2$	$= ½G.M.m / R$	[J]	E_{pot} at surface converted into E_{kin} at core
v^2	$= G.M / R$	[$m^2.s^{-2}$]	
v	$= (G.M / R)^{½}$ = 7,910	[$m.s^{-1}$]	

The pebble reaches a staggering 7,910 [$m.s^{-1}$] at the core of the earth, which equals more than 28,000 kilometers per hour, (about 8 km / second) which does serious damage to anything that might obstruct the pebble at the core. Note that all mass-particles, pebbles, and hydrogen molecules reach the same speed at the core when starting at zero speed at the surface. However, do not forget that the density within the earth is not the same everywhere, a precondition to formula (2). In other words, these computations use a simplified model of the earth, creating an additional error.

Thermal equilibrium

Thermal equilibrium is reached when as much radiation power in [W] is received at the surface as is radiated. If the surface temperature remains constant, than so does the core temperature. Earth's surface temperature has remained relatively constant over the last million years, thus so has the core temperature. This means that the radioactive decay does not contribute to the core temperature in the current thermal equilibrium, which goes against intuition. To further strengthen our case that energy conservation determines core temperature, let us look at the core temperature of the sun.

The core temperature of the sun comes to 15,550,000 [K] based on energy conservation in the same way as we did for the earth; a temperature which is confirmed by astrophysicists. In other words, the core temperature is not *caused* by nuclear fusion, but the nuclear fusion is *the result* of the high temperature caused by energy conservation within an object which is in thermal equilibrium.

Conclusion

Noether's law of energy conservation explains the core temperature of spherical objects in thermal equilibrium. Radioactive decay or nuclear fusion create high amounts of power. However, an object in thermal equilibrium transports as much power to the surface into the surroundings, neutralizing the generated power. The core temperature of the earth must be at least 5.380 [K]; the core temperature of the sun must be at least 15,550,000 [K]. The core temperature of spherical masses is mainly caused by energy conservation.

Consequences

Noether's law of energy conservation puts a low limit to the mass "M" of spherical objects existing out of gasses (hydrogen and helium) in order to ignite the fusion process. Gas planets like Jupiter and Saturn do not have enough mass to ignite the fusion process.

About our book "Quantum Relativity (for Gravitation)"

Emmy Noether is the mother of energy-momentum conservation. The Schwarzschild solution to Einstein's theory of General Relativity upon which a massive sphere is modelled, needs a small repair to abide by her conservation laws. The authors call their *repaired* Schwarzschild solution "Quantum Relativity for Gravitation", see www.loop-doctor.nl/book-ii, of which you can read the first three chapters. The accomplishments of our theory are:

1. Uniting Special Relativity with the Schwarzschild solution,
2. Making the Schwarzschild solution comply with Noether's conservation laws.
3. Simplifying the Schwarzschild solution ($E = \sigma.\gamma.m_0.c^2$)

Quantum Relativity for Gravitation predicts that geologists will, at some point in the future, be able to confirm that the high temperature at the core of the earth is *mainly* caused by its position at the center-of-mass (energy conservation). We hope you get as many "aha" experiences as we did,

Rob Roodenburg (MSc, author)
Frans de Winter (MSc. coauthor)
Oscar van Duijn (MSc. coauthor)
Maarten Palthe (MSc. editor)

Schiedam,
February 2017