Relativity Solutions need repair

Introduction and relevance

Did you know that there are five *different* "Schwarzschild Solutions" (two of Karl Schwarzschild, one of Droste/Eddington, one of Misner, Thorne, and Wheeler, and one of Kip Thorne)? There is no such thing as *the* Schwarzschild Solution. So, how do you know which one is right? Was Karl Schwarzschild right about his *two* solutions (one of a mass-point and one of a sphere of incompressible liquid)? Was Kip Thorne right about his solution based on "Tortoise coordinates"? Is a different solution, like the Robertson-Walker Solution, correct? The authors present you with three ways to check the outcome, based on:

1. The principles of General Relativity,
2. The Laplace operator of General Relativity,
3. The field equations of the solution of General Relativity.

These checks are based on the documents produced by Albert Einstein in the "Annalen der Physik" number 49 of 1916.

Additionally we will check the outcome for Noether's theorem; Emmy Noether pointed out that Einstein's theory of General Relativity are in some respects contradicting her theorem. The outcome is disappointing, none of the considered solutions is 100% correct!

1. Checking the outcome against the principles of General Relativity

Einstein states: "… for ds^2 is a quantity ascertainable by rod-clock measurement of point-events infinitely proximate in space-time, and defined independently of any particular choice of coordinates" [1]). This is also called the "Relativity Principle".

How does this help us to check the outcome of a solution? The outcome can be checked with a local (cesium) clock measuring a local time difference "dt_0", called the proper time difference. Or we can simply compare the outcome before and after the coordinate transformation.
In Einstein's theory, the speed of light "c" is unified (c = 1), making "ds" equal to "dt_0". for those scientists that want to keep the speed of light equal to its SI definition, we can state that $ds = c.dt_0$.

This entails the possibility to change from a Cartesian space-time coordinate system (t,x,y,z) to another Cartesian coordinate system (t',x',y',z') provided the measured proper time "dt_0" remains the same. For example the "space axes transformation":

t	= t	[s]	time transformation (staying the same)
x'	= y	[m]	first coordinate axis transformation
y'	= z	[m]	second coordinate axis transformation
z'	= x	[m]	third coordinate axis transformation

Note that this transformation requires the same units of measurement in both coordinate systems. If the units in which (t',x',y',z') is measured are twice as large, the resulting proper time is half as much as computed in (t,x,y,z). When using another set of units than the SI system on earth at sea-level, one has to make sure that the product of the coordinate and its unit of measurement is used (uniform measurement principle) instead of coordinate-only formulas, see appendix.

So, the first way of checking a solution is to apply the simple "space axes transformation" as described. If the outcome of "ds" (= $c.dt_0$) is different, the solution does not check out!

2. Checking the outcome against Einstein's Laplace operator

Einstein makes the connection between the covariant metric tensor element g_{44} (nowadays g_{00}) and the gravitational potential of a sphere of incompressible liquid "Φ" [2]):

$\Delta g_{44} = \Delta\Phi = 8\pi.G.\rho / c^2$ $[m^{-2}]$ Laplace operator of sphere of incompressible liquid (1)

The Laplace operator is the sum of the three space coordinate second order partial differentials, see appendix. The gravitational potential "Φ" outside of the sphere (in vacuum) is $1 - R_S / r$, according to Einstein [2])

How that helps us to check a solution? The theory of General Relativity is based on this formula! Any gravitational potential obtained, must abide by this formula. One thing you can use it for, is checking a formula for the gravitational potential inside of the sphere of incompressible liquid as Misner, Thorne, and Wheeler and also Karl Schwarzschild worked out.

3. Checking the outcome against the field equations

Solutions have from 2 to 10 elements of the covariant metric tensor "g" which are unequal to zero (these do no "vanish"). The Shapiro solution has just two elements, the Schwarzschild solution has four elements, and Einstein's first "g" has 6 elements unequal to zero of which two are the same, see our book II. The Shapiro solution has just two field equations, the Schwarzschild solution has four field equations, and Einstein's "g" has five field equations, exactly equal to the number of unknown elements.

Since the number of unknowns is equal to the number of equations, the field equations are solvable. When a solution is found, the solution must be put back into the field equations and the same outcome on both sides of the "=" sign should appear. If so, the solution is correct according to Einstein's theory of General Relativity.

Checking the outcome with Noether's theorem

The key question remains whether Einstein's theory of General Relativity is an exact description of the gravitational field. Emmy Noether was not convinced. Emmy's criticism about Albert's theory is found in 1918 in her thesis [3]): "Hilbert enunciates his assertion to the effect that the failure of proper laws of conservation of energy is a characteristic feature of the general theory of relativity". Although written by Emmy, she could not be a professor as a woman, so the correspondence was officially done (and supported) by professor Hilbert.

Noether's theorem requires the speed of light "c" in vacuum and other constants of nature to be the same everywhere and all the time within the defined reference frame. The speed of light within a reference frame in vacuum can simply be obtained as ds / dt, the distance travelled divided by the time taken. Do not confuse proper time difference "*ds*" or "dt_0" with space distance "ds".

How this helps to check a solution for Noether's theorem? The answer is simple, both "ds" and "dt" are found in all published solutions to Einstein's theory of General Relativity We know that light in vacuum travels with exactly "c" in all directions, while the proper time difference "dt_0" is zero. For example, when we look at the generic Schwarzschild solution and move a mass-particle in the x direction (ds = dx), see appendix, we obtain:

$c^2.dt_0^2 = 0 = W(r).c^2.dt^2 - X(r).dx^2$ $[m_0^2]$
$ds^2/dt^2 = dx^2 / dt^2 = \{W(r) / X(r)\}.c^2$ $[m^2.s^{-2}]$
$v = \{W(r) / X(r)\}^{½}.c$ $[m.s^{-1}]$

If W(r) is unequal X(r), then the speed of light is unequal "c" and the solution does not abide by Noether's theorem.

The Solutions checked

The authors have checked all the (five) Schwarzschild, Shapiro, and Robertson-Walker solutions, with very disappointing results. None of the five Schwarzschild solutions abides by the first check (the space axes coordinate transformation). None of the solutions abides by the second check (Einstein's Laplace operator), except the Robertson-Walker solution. None of the solutions has checked out for the third check (all field equations matching the outcome). The separate check on Noether's theorem is equally disappointing, no solution can claim energy and momentum conservation.

Summary

Einstein's theory of General Relativity is unfinished and needs to be repaired for Einstein's own and ignored principles and Noether's theorem. The authors have managed to find a new time-like solution inside of a sphere of incompressible liquid based on the second check, and repaired the Schwarzschild and the Robertson-Walker solution for Noether's theorem. This 15 years of work leads to surprising results. Reality is simpler, more elegant and mystic assumptions disappear.
For more information, see www.loop-doctor.nl.

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[1])"The Foundation of the General Theory of Relativity" of 1916. Paragraph 4, (translated).
[2])"The Foundation of the General Theory of Relativity", Paragraph 21 formulas (67, 68).
[3]) Tavel. M. "Invariant Variation Problems", Transport Theory and Statistical Physics 1971, 1(3) p. 186-207.

Appendix

Minkowski formula within a Euclidean reference frame:

$ds^2 = c^2.dt_0^2 = c^2.dt^2 - dx^2 - dy^2 - dz^2$ $[m_0^2]$ Euclidean
$c^2.dt_0^2.[m_0^2] = c^2.dt^2.[m^2] - dx^2.[m^2] - dy^2.[m^2] - dz^2.[m^2]$ uniform Minkowski
$c^2.dt_0^2.[m_0^2] = c^2.dt^2.[m^2] - ds^2.[m^2]$ uniform Minkowski

Einstein's Laplace operator "Δ" of the gravitational potential "Φ" of a sphere of incompressible liquid:

$R_S = 2G.M / c^2$ [m] Schwarzschild radius
$\Phi = 1 - R_S / r$ [ ] gravitational potential
$\Delta\Phi = (\partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2)\Phi = 8\pi.G.\rho / c^2$ $[m^{-2}]$ Laplace operator of "Φ"

General format of the Schwarzschild Solutions to a sphere of incompressible liquid:

$ds^2 = c^2.dt_0^2 = W(r).c^2.dt^2 - X(r).dx^2 - Y(r).dy^2 - Z(r).dz^2$ $[m_0^2]$ Cartesian
$ds^2 = c^2.dt_0^2 = W(r).c^2.dt^2 - X(r).r^2.\sin^2\theta.d\varphi^2 - Y(r).r^2.d\theta^2 - Z(r).dr^2$ $[m_0^2]$ Polar

$dt = dt$ [s]
$dx = r.\sin\theta.d\varphi$ [m]
$dy = r.d\theta$ [m]
$dz = dr$ [m]

$ds^2 = dx^2 + dy^2 + dz^2 = r^2.\sin^2\theta.d\varphi^2 + r^2.d\theta^2 + dr^2$ $[m^2]$

W(r), X(r), Y(r), and Z(r) are functions of the distance "r" to the center-of-mass of a sphere of incompressible liquid.