

The day Relativity changed

Introduction

Einstein published his theory of General Relativity (GR) in several articles in the “Analen der Physik”. The last one (translated): “The Foundation of the General Theory of Relativity”¹ is the *official* GR, published on the 11th of May 1916. This date is crucial in understanding the Schwarzschild Solutions, since Karl Schwarzschild published two exact solutions to the *unofficial* GR in January and February 1916, before Einstein’s official publication of GR. Karl Schwarzschild died on the day that Einstein’s GR was published, on that same 11th of May 1916. Moreover, on that same day Einstein changed his theory of Special Relativity (SR) *officially*, after criticism from Mach, Lorentz, and Ehrenfest on his earlier version of 1905. Einstein’s Relativity changed significantly on that 11th of May 1916.

Relevance to SR

The change in SR is very relevant and significant; it is the change from two *equal* reference frames in which all the laws of nature are valid in both frames, to a travelling mass-point within a reference frame in which all the laws of nature are valid within the reference frame only. It is the change from two equal reference frames in the Lorentz transformation to a mass-point within one reference frame according to Minkowski’s space-time. This is all described in Einstein’s *official* GR in the paragraphs 2, 3, and 22, culminating in formula (1) in paragraph 4. This changes the equality of reference frames into the dominance of reference frames with “ponderable masses” after formula (72) of paragraph 22.

With this change in SR the twin paradox is solved; the universal reference frame is dominant over all other local reference frames, whose clocks all tick slower than the universal clocks. The travelling twin at high speed within the universe is much younger than her twin upon return on earth. It also solves the Ehrenfest paradox, which is a consequence of the Lorentz transformation. All paradoxes disappear in Einstein’s SR of 1916! It shows Einstein’s social agility to hide such a major change in his official GR, knowing that very few people would read all 22 paragraphs. He once joked: “two things are infinite, space-time and the stupidity of mankind, but I am not so sure of space-time”.

Relevance to the Schwarzschild Solution

Karl’s *exact* solutions are (translated): “On the Gravitational Field of a Mass-point” and “On the Gravitational Field of a Sphere of Incompressible Liquid”. These solutions are thus based on Einstein’s *unofficial* GR. Einstein’s official GR does not invalidate Karl’s solutions, but opens up a can of worms in Relativity. From that 11th of May 1916 onwards, a crucial requirement of his earlier version is made *optional*: $g = -1^2$, see footnote 2. Having an optional requirement makes solutions to his field equations *undetermined* if one chooses *not* to make use of the optional $g = -1$ condition. This is similar to having two unknown variables and only one equation: many solutions can be found.

Consequently, since that 11th of May 1916 multiple “Schwarzschild Solutions” have appeared, the latest with Eddington/Finkelstein coordinates to “fall through the event horizon”. This was made possible by Einstein by making $g = -1$ *optional*. Unfortunately, this has led to singularities in

¹ Einstein A. (translated) “The Foundation of the General Theory of Relativity”, Annalen der Physik 49, 1916

² Paragraph 19 footnote at the end: “On the abandonment of the choice of coordinates with $g = -1$...”

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physics (opposed to singularities in mathematics), like the *presumed* “black hole singularity with event horizon”. Amazingly, the optional $g = -1$ creates a problem when you *do choose* to use the option (as Karl Schwarzschild did), but also when you *choose not* to use the option (Robertson-Walker)! Let us explain the essence of the problem: energy-momentum conservation according to Noether’s theorems.

Relevance to GR

Emmy Noether warned Einstein about the lack of *guaranteed* energy conservation in his GR in her publication “Invariant variation problems” of 1918. The problem is as follows: when you choose to use the $g = -1$ option, the speed of light is not exactly “c” in all directions. In the non-radial directions of the Schwarzschild Solutions, the speed of light is *unequal* to “c”, while being exactly equal to “c” is a requirement to Noether’s theorems. Einstein neglected this because within our solar system, the effect is immeasurably small. However, applied to the strong gravitation around black holes, it becomes very relevant! When you choose *not* to use the $g = -1$ option, GR is undetermined and no guarantee of energy and momentum conservation can be given. In other words, GR is *unfinished* if energy and momentum guarantees are looked for.

Noether’s theorems in GR Solutions

The optional $g = -1$ condition has to change into a condition which ensures that the solutions are abiding by Noether’s theorems of energy and momentum conservation. This is not hard to do for the Schwarzschild and Robertson-Walker (R&W) solutions, but it is much harder to do for the Kerr solution, frame dragging, and other solutions that do not have a diagonal covariant metric tensor “ g_{ii} ”. To demonstrate the Schwarzschild Solution repaired for Noether’s theorem, you get:

$$\begin{aligned}\Phi_{\text{inside}} &= 1 - 1/2Rs / R + 1/2Rs.r^2 / R^3 & [] & \text{gravitational potential inside } (r \leq R) \\ \Phi_{\text{outside}} &= 1 - Rs / r & [] & \text{gravitational potential outside } (r \geq R)\end{aligned}$$

For the full derivation see our book “Repairing Schwarzschild’s Solution”. In these equations is “Rs” the Schwarzschild radius, “R” the radius of the sphere and “r” the distance from the center-of-mass of the equal-density sphere. Note that the outcomes at the surface of the sphere ($r = R$) are the same. Also note that at $r = 0$, the gravitational potential is $1 - 1/2Rs / R$. A gravitational potential is a number between zero and one. At zero, space would curve into itself. At one, there is no gravitational field. So at any sphere within the universe $0 < \Phi < 1$.

Conclusions and more information

This leads to $R \geq 1/2Rs$. In other words, a constant density sphere has no “event horizon” at $r = Rs$. In the *repaired* R&W solution, the speed of light is the same in all directions and at all times, while Einstein’s SR of 1916 becomes a special case of the solution. The universe does not need “dark energy”, see our book “Repairing Robertson-Walker’s Solution”.

Our three books and articles on www.loop-doctor.nl describe the repair of Einstein’s Relativity for Noether’s theorems³ in full detail. We hope you get as many “aha” experiences as we did,

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³ Noether E. “Invariant variation problems” translated by Tavel M. TTSP 1971 p. 186-207